# Online Appendix

## Omitted Proofs of Lemmas

**Proof of Lemma 3.** We begin by showing the first point above. The second point follows by definition of $\alpha_n(p)$.

First, suppose by contradiction that there exists a $(p, n)$ on-path such that $\alpha_n(p) < \min\{\beta(1-p)/k_n, 1\}$. Recalling that $p(s)$ is given by (2), I begin by showing that for all $s$ sufficiently small, $(p(s), n)$ is on-path. Suppose not by contradiction. Since $(p, n)$ is on-path, this implies that $F_{p,n}(s) = 1$, which contradicts Lemma 1. It thus follows from (4), combined with the piecewise twice differentiability and right-differentiability of $F_{p,n}$, that $\alpha_n(p(s))$ is continuous in some right-neighborhood of $s = 0$. Thus, there exists an $\varepsilon > 0$ such that for all $s \in [0, \varepsilon]$, $k_n\alpha_n(p(s)) < \beta(1-p)$.

Next, I claim that $F_{p,n}(\varepsilon) > 0$. Suppose this is not true by contradiction. Then, it follows that $F_{p,n}(s) = 0$ for all $s \in [0, \varepsilon]$, implying by definition of $\alpha$ that $\alpha_n(p) = 1$, contradicting the assumption that $\alpha_n(p) < 1$. Now, define the following deviation $\tilde{F}_{p,n}$, which shifts the mass $F_{p,n}$ places on $[0, \varepsilon]$ to $\infty$:

$$\tilde{F}_{p,n}(s) = \begin{cases} 0 & \text{if } s \in [0, \varepsilon] \\ F_{p,n}(s) - F_{p,n}(\varepsilon) & \text{if } s \in (\varepsilon, \infty) \\ 1 & \text{if } s = \infty. \end{cases}$$

The admissibility (i.e., right-continuity and piecewise twice-differentiability) of $\tilde{F}_{p,n}$ follows from the admissibility of $F_{p,n}$. We now wish to show that $\tilde{F}_{p,n}$ is a profitable deviation at $(p, n)$. Let $\Psi$ denote the first-report distribution under the strategy profile where all players play $F_{p,n}$, and let $\tilde{\Psi}$ denote the first-report distribution under the strategy profile where $i$ plays $\tilde{F}_{p,n}$ and all $j \neq i$ play $F_{p,n}$.

By definition of $\Psi$, $\tilde{\Psi}^i(s) = \Psi^i(s) - X(s)$, where

$$X(s) = \begin{cases} p\int_0^s e^{-\lambda r(N-n)}(1 - F_{p,n}(r))^{N-n} d(e^{-\lambda r}(F_{p,n}(r) - 1)) & \\ +(1-p)\int_0^s (1 - F_{p,n}(r))^{N-n} dF_{p,n}(r) & \text{if } s \in [0, \varepsilon] \\ X(\varepsilon) & \text{if } s > \varepsilon. \end{cases}$$

$X(s)$ is weakly increasing in $s$. Furthermore, because $F_{p,n}(\varepsilon) > 0$, it follows that $F_{p,n}(s)$ strictly increases on $[0, \varepsilon]$. Thus, $X(s)$ is strictly increasing at some $s \in [0, \varepsilon]$.

Now, by the above definition:

$$\int_0^\infty [k_n\alpha_n(p(s)) - \beta(1-p^i(s))]d\tilde{\Psi}^i(s) - \int_0^\infty [k_n\alpha_n(p(s)) - \beta(1-p^i(s))]d\Psi^i(s)$$
$$= -\int_0^\varepsilon [k_n\alpha_n(p(s)) - \beta(1-p(s))]dX(s) > 0.$$

where the strict inequality follows from the fact that $X(s)$ is strictly increasing on $[0,\varepsilon]$ and the above-established fact that $k_n\alpha_n(p(s)) < \beta(1-p(s))$ for all $s \in [0,\varepsilon]$.

Next, let us consider $\tilde{\Psi}^{-i}(s)$. By definition, $\tilde{\Psi}^{-i}(s) = \Psi^{-i}(s) - Y(s)$, where

$$Y(s) = -p\int_0^s [e^{-\lambda r}(1-F_{p,n}(r))]^{n-2}F_{p,n}(\min\{r,\varepsilon\})d(e^{-\lambda r}(F_{p,n}(r)-1)) -$$
$$(1-p)\int_0^s (1-F_{p,n}(r))^{n-2}F_{p,n}(\min\{r,\varepsilon\})dF_{p,n}(r).$$

Thus,

$$\int_0^\infty V_{p^{-i}(s),n+1}d\tilde{\Psi}^{-i}(s) - \int_0^\infty V_{p^{-i}(s),n+1}d\Psi^{-i}(s) = \int_0^\infty V_{p^{-i}(s),n+1}dY(s) \geq 0.$$

where the final inequality follows from the fact that $Y(s)$ is weakly increasing in $s$ and $V_{p^{-i}(s),n+1} \geq 0$. Combining the previous two inequalities, we obtain that $V_{p,n}(\tilde{F}_{p,n}) > V_{p,n}(F_{p,n})$, and thus $i$ can profitably deviate at $(p,n)$. Contradiction. □

**Proof of Lemma 4.** Fix a $(p,n)$ on-path. I first show that for all $s \geq 0$,

$$\alpha_n(p(s)) = \frac{\lambda p(s)}{\lambda p(s) + \frac{F'_{p,n}(s+)}{1-F_{p,n}(s)}} \tag{22}$$

It follows from Lemma 3 that $(p(s),n)$ is on-path for all $s \geq 0$. Thus, by Lemma 1, $F_{p(s),n}(0) = 0$, and by (4) $\alpha_n(p(s)) = \frac{\lambda p(s)}{\lambda p(s) + F'_{p(s),n}(0+)}$. Next, it follows from (3) that $F'_{p(s),n}(0+) = \frac{F'_{p,n}(s+)}{1-F_{p,n}(s)}$. Combining the previous two equations yields (22). It thus follows from the right-differentiability and piecewise twice-differentiability of $F_{p,n}$ that $\alpha_n(p(s))$ is right-continuous in $s$. It remains to show that $\alpha_n(p(s))$ is left-continuous in $s$. Suppose by contradiction there exists an $s$ such that $\alpha_n(p(s))$ is left-discontinuous. Then there exists some $d > 0$ such that for all $\varepsilon > 0$, there exists an $s_\varepsilon \in (s-\varepsilon, s)$ such that $|\alpha_n(p(s_\varepsilon)) - \alpha_n(p(s))| > d$. First consider the case where for all $\varepsilon > 0$, there exists an $s_\varepsilon \in (s-\varepsilon, s)$ such that $\alpha_n(p(s_\varepsilon)) - \alpha_n(p(s)) > d$. I begin by claiming that for all $\varepsilon > 0$,

$$V_{p(s_\varepsilon),n} = V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon}). \tag{23}$$

Note that there exists some $s^* \in (s, \infty]$ such that $V_{p(s_\varepsilon),n} = V_{p(s_\varepsilon),n}(\delta_{s^*-s_\varepsilon})$. To see why this must hold, suppose not, by contradiction. Then it must be that $F_{p(s_\varepsilon),n}$ places full mass on $[0, s - s_\varepsilon]$, and thus, either Lemma 1 or (3) would be violated. Thus, we have

$$\begin{aligned}
V_{p(s_\varepsilon),n} &= \int_0^{s-s_\varepsilon} k_n\alpha_n(p(s_\varepsilon + r))d\Psi^i(r) + (N-n)\int_0^{s-s_\varepsilon} V_{p^i(s_\varepsilon+r),n+1}d\Psi^{-i}(r) \\
&\quad + (1 - \sum_j \Psi^j(s - s_\varepsilon))V_{p(s),n}(\delta_{s^*-s}) \\
&= \int_0^{s-s_\varepsilon} k_n\alpha_n(p(s_\varepsilon + r))d\Psi^i(r) + (N-n)\int_0^{s-s_\varepsilon} V_{p^i(s_\varepsilon+r),n+1}d\Psi^{-i}(r) \\
&\quad + (1 - \sum_j \Psi^j(s - s_\varepsilon))V_{p(s),n}(\delta_0) = V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon}),
\end{aligned}$$

where $\Psi$ is the first-report distribution associated with the strategy profile in which $i$ plays $\delta_\infty$ and all $j \neq i$ play $F_{p(s_\varepsilon),n}$. Note that the equality follows from the fact that $\alpha_n(p(s)) < 1$, and thus by Lemma 2, $V_{p(s),n} = V_{p(s),n}(\delta_0)$. However, note that for all $\varepsilon > 0$,

$$\begin{aligned}
V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon}) = \int_0^{s-s_\varepsilon} & k_n\alpha_n(p(s_\varepsilon + r))d\Psi^i(r) + (N-n)\int_0^{s-s_\varepsilon} V_{p^i(s_\varepsilon+r),n+1}d\Psi^{-i}(r) \\
& + (1 - \sum_j \Psi^j(s - s_\varepsilon))[k_n\alpha_n(p(s), n) - \beta(1 - p(s))].
\end{aligned}$$

Because the $\Psi^j$ are absolutely continuous,

$$\lim_{\varepsilon \to 0} V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon}) = k_n\alpha_n(p(s), n) - \beta(1 - p(s)).$$

Then, by the assumption that $\alpha_n(p(s_\varepsilon)) - \alpha_n(p(s)) > d$, for all $\varepsilon > 0$ sufficiently small $V_{p(s_\varepsilon),n}(\delta_0) = k_n\alpha_n(p(s_\varepsilon), n) - \beta(1 - p(s_\varepsilon)) > V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon})$, contradicting (23).

Next, consider the case where for all $\varepsilon > 0$, $\alpha_n(p(s)) - \alpha_n(p(s_\varepsilon)) > d$. As shown above, $\lim_{\varepsilon \to 0} V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon}) = V_{p(s),n}(\delta_0)$. Thus, for $\varepsilon$ sufficiently small,

$$V_{p(s_\varepsilon),n}(\delta_{s-s_\varepsilon}) > k_n\alpha_n(p(s_\varepsilon)) - \beta(1 - p(s_\varepsilon)) = V_{p(s_\varepsilon),n}(\delta_0).$$

However, since $\alpha_n(p(s_\varepsilon)) < 1$ for all $\varepsilon > 0$, by Lemma 2, $V_{p(s_\varepsilon),n} = V_{p(s_\varepsilon),n}(\delta_0)$. Contradiction. $\square$

**Proof of Lemma 5.** Fix an $(\alpha, F)$. I begin by establishing the necessity of the three conditions specified in Definition 3 for $(\alpha, F)$ to be an equilibrium. First we establish the necessity of part 3. of Definition 3. To this end, recall that by the selection assumption, $F_{1,n}(0) = 1$. Thus, it follows from (4) that $\alpha_n(1) = 0$ if $(p = 1, n)$ is on-path. Parts 1. and 2. of Definition (3) follow immediately from Proposition 1 and Proposition 2, respectively.

Next, we establish the sufficiency of the above conditions for $(\alpha, F)$ to be an equilibrium. We begin by considering the case in which $k_n < \beta$ and $p \leq p_n^*$. It follows from Definition 3 that $\alpha_n(q) = 1$ for all $q \leq p$. Thus, by (4), $F_{p,n} = \delta_\infty$. We want to show that there exist no profitable deviations in this case, i.e., that $V_{p,n} = V_{p,n}(\delta_\infty)$. It suffices to show that

$$V_{p,n}(\delta_\infty) \geq V_{p,n}(\delta_s) \text{ for all } s \in [0, \infty). \tag{24}$$

First, note that for all $s \in (0, \infty)$,

$$V_{p,n}(\delta_s) = k_n(1 - p(1 - e^{-\lambda s(N-n+1)})(\frac{N-n}{N-n+1})) - \beta(1-p) \leq k_n - \beta(1-p) = V_{p,n}(\delta_0).$$

Further, $k_n \leq \beta$ and $p \leq p_n^*$ implies that $V_{p,n}(\delta_0) = k_n - \beta(1-p) \leq \frac{k_n}{N-n+1} = V_{p,n}(\delta_\infty)$. Thus, $V_{p,n}(\delta_\infty) \geq V_{p,n}(\delta_s)$ for all $s \in [0, \infty)$.

Next, we show that $F_{p,n}$ is optimal when $k_n > \beta$ or $p \geq p_n^*$. We begin by showing

$$\frac{d}{d\Delta} V_{p,n}(\delta_\Delta) = 0 \text{ for all } \Delta \in [0, \infty) \text{ if } k_n \geq \beta \text{ and for all } \Delta \in [0, t^*) \text{ if } k_n < \beta \tag{25}$$

where $t^*$ is the unique solution to $p(t^*) = p_n^*$. Note that

$$\begin{aligned} V_{p,n}(\delta_\Delta) = \int_0^\Delta k_n \alpha_n(p(s)) d\Psi^i(s) + (N-n) \int_0^\Delta V_{p^i(s),n+1} d\Psi^{-i}(s) + \\ (1 - \sum_j \Psi^j(\Delta))(k_n \alpha_n(p(\Delta)) - \beta(1 - p(\Delta))), \end{aligned} \tag{26}$$

where $\Psi$ is the first-report distribution associated with the strategy profile in which $i$ plays $\delta_\infty$ and all $j \neq i$ play $F_{p,n}$. Then,

$$\frac{d}{d\Delta} V_{p,n}(\delta_\Delta) = (N-n)[V_{p^i(\Delta),n+1} - k_n \alpha_n(p(\Delta)) + \beta(1 - p(\Delta))]\Psi^{-i\prime}(\Delta) - \beta(1 - p(\Delta))\Psi^{i\prime}(\Delta)$$
$$+ (1 - \sum_j \Psi^j(\Delta)) p'(\Delta)(k_n \alpha_n'(p(\Delta)) + \beta),$$

where $\Psi^{i\prime}(t) \equiv \frac{d}{dt}\Psi^i(t)$.

In the above, the existence of $\Psi^{j\prime}(\Delta)$ follows from the differentiability of $\alpha_n$ at $p(\Delta)$, and thus, the differentiability of $F_{p,n}$ at $\Delta$. We wish to show that $\frac{d}{d\Delta}V_{p,n}(\delta_\Delta) = 0$. To this end, we begin by deriving expressions for $\Psi^{i\prime}(\Delta)$ and $\Psi^{-i\prime}(\Delta)$. First, it follows by definition of the first-report distribution that:

$$\Psi^i(\Delta) = p\lambda \int_0^\Delta (1 - F_{p,n}(s))^{N-n} e^{-\lambda(N-n+1)s} ds.$$

Differentiating this, we obtain:

$$\Psi^{i\prime}(\Delta) = p\lambda(1 - F_{p,n}(\Delta))^{N-n} e^{-\lambda(N-n+1)\Delta}.$$

Meanwhile:

$$\Psi^{-i}(\Delta) = p \int_0^\Delta (1-F_{p,n}(s))^{N-n-1} e^{-\lambda(N-n)s} d((F_{p,n}(s)-1)e^{-\lambda s}) + (1-p) \int_0^\Delta (1-F_{p,n}(s))^{N-n-1} F'_{p,n}(s) ds.$$

where the existence of $F'_{p,n}(s)$ again follows from the assumption that $\alpha_n$ is differentiable at $p(s)$. Differentiating this, we obtain:

$$\Psi^{-i\prime}(\Delta) = (1 - F_{p,n}(\Delta))^{N-n}[\frac{F'_{p,n}(\Delta)}{1 - F_{p,n}(\Delta)}(pe^{-\lambda\Delta(N-n+1)} + (1-p)) + pe^{-\lambda\Delta(N-n+1)}\lambda].$$

It follows from (4) and (3) that

$$\frac{F'_{p,n}(\Delta)}{1 - F_{p,n}(\Delta)} = \lambda p(\Delta)(\frac{1}{\alpha_n(p(\Delta))} - 1).$$

Substituting this, along with the definition of $p(\Delta)$, we obtain:

$$\Psi^{-i\prime}(\Delta) = \lambda(1 - F_{p,n}(\Delta))^{N-n}(pe^{-\lambda\Delta(N-n+1)} + (1-p))\frac{p(\Delta)}{\alpha_n(p(\Delta))}.$$

Note further that

$$1 - \sum_j \Psi^j(\Delta) = (1 - F_{p,n}(\Delta))^{N-n}(pe^{-\lambda\Delta(N-n+1)} + (1-p)). \tag{27}$$

Substituting the expressions for $\Psi^{i\prime}(\Delta)$, $\Psi^{-i\prime}(\Delta)$, and $1 - \sum_j \Psi^j(\Delta)$ into the expression for $\frac{d}{d\Delta}V_{p,n}(\delta_\Delta)$ we obtain:

$$\frac{d}{d\Delta}V_{p,n}(\delta_\Delta) = K[\frac{(N-n)}{\alpha_n(p(\Delta))}(V^i_{p(\Delta),n+1} - k_n\alpha_n(p(\Delta)) + \beta(1-p(\Delta))(1-\alpha_n(p(\Delta)))) \\ -k_n\alpha_n'(p(\Delta))(1-p(\Delta))(N-n+1)].$$

where $K \equiv \lambda(1-F_{p,n}(\Delta))^{N-n}(pe^{-\lambda\Delta(N-n+1)}+(1-p))p(\Delta)$. Because (ODE) is satisfied at $(p(\Delta),n)$, using it to substitute in for $\alpha_n'(p(\Delta)$, we obtain (25).

Now, consider the case where $k_n \geq \beta$. To show $F_{p,n}$ is optimal, it suffices to show that all pure strategies $\delta_\Delta$ yield the same payoff, i.e., that

$$V_{p,n}(\delta_0) = V_{p,n}(\delta_\Delta) \tag{28}$$

for all $\Delta \in [0,\infty]$. It follows from (25) that (28) holds for all $\Delta \in [0,\infty)$. It remains to show that (28) holds for $\Delta = \infty$. By (25),

$$\begin{aligned} &V_{p,n}(\delta_0) = \lim_{\Delta\to\infty} V_{p,n}(\delta_\Delta) \\ &= \lim_{\Delta\to\infty}\int_0^\Delta k_n\alpha_n(p(s))d\Psi^i(s) + (N-n)\lim_{\Delta\to\infty}\int_0^\Delta V_{p^i(s),n+1}d\Psi^{-i}(s)+ \\ &\lim_{\Delta\to\infty}(1-\sum_j \Psi^j(\Delta))(k_n\alpha_n(p(\Delta)) - \beta(1-p(\Delta))) \\ &= \int_0^\infty k_n\alpha_n(p(\Delta))d\Psi^i(s) + (N-n)\int_0^\infty V_{p^i(\Delta),n+1}d\Psi^{-i}(s) = V_{p,n}(\delta_\infty), \end{aligned}$$

where the third equality follows from the limit condition $\lim_{p\to 0+}\alpha_n(p) = \beta/k_n$.

Finally, consider the case where $k_n < \beta$ and $p > p_n^*$. Because $\alpha_n(p(s)) = 1$ for all $s > t^*$, by (4), it follows that $F'_{p,n}(s) = 0$ for all $s > t^*$. Thus, the support of $F_{p,n}$ is a subset of $[0,t^*]\cup\infty$. Thus, to show $F_{p,n}$ is optimal, it suffices to show that $\delta_\Delta$ is optimal for $\Delta \in [0,t^*]\cup\infty$. I first show that

$$V_{p,n}(\delta_\Delta) = V_{p,n}(\delta_0) \text{ for all } \Delta \in [0,t^*]\cup\infty \tag{29}$$

and then show

$$V_{p,n}(\delta_{t^*}) \geq V_{p,n}(\delta_\Delta)\text{for all } \Delta \in (t^*,\infty). \tag{30}$$

To show (29), recall that it follows from (25) that

$$V_{p,n}(\delta_0) = V_{p,n}(\delta_\Delta) \text{ for all } \Delta \in [0,t^*).$$

It remains to show $V_{p,n}(\delta_0) = V_{p,n}(\delta_s)$ for $s \in \{t^*, \infty\}$. For $s = t^*$, it follows from the above that

$$V_{p,n}(\delta_0) = \lim_{\Delta \to t^*-} V_{p,n}(\delta_\Delta) = V_{p,n}(\delta_{t^*}),$$

where the final inequality follows from (26), and the continuity of $\alpha_n(p(t))$ and $\Psi^j$ at $t^*$. I will now show $V_{p,n}(\delta_{t^*}) = V_{p,n}(\delta_\infty)$. Note that for all $\Delta \in [t^*, \infty]$:

$$V_{p,n}(\delta_\Delta) = \int_0^{t^*} k_n \alpha_n(p(s)) d\Psi^i(s) + (N-n) \int_0^{t^*} V_{p^i(s),n+1} d\Psi^{-i}(s) + (1 - \sum_j \Psi^j(t^*)) V_{p_n^*,n}(\delta_{\Delta - t^*}).$$

Thus, to show $V_{p,n}(\delta_{t^*}) = V_{p,n}(\delta_\infty)$, it suffices to show $V_{p_n^*,n}(\delta_0) = V_{p_n^*,n}(\delta_\infty)$. It follows from the definition of $p_n^*$ that:

$$V_{p_n^*,n}(\delta_0) = k_n - \beta(1 - p_n^*) = \frac{k_n p_n^*}{n} = V_{p_n^*,n}(\delta_\infty).$$

Similarly, to show (30), it suffices to show that $V_{p_n^*,n}(\delta_0) \geq V_{p_n^*,n}(\delta_\Delta)$ for all $\Delta \in (0, \infty)$, which we have established in (24). □

**Proof of Lemma 6.** Let $\hat{V}_{p,n}(\delta_t)$ denote the value from $\delta_t$ under state $(p, n)$ (i.e., under common belief $p$) assuming the firm is informed and thus holds belief 1. To prove the above statement, it suffices to show

$$\hat{V}_{p,n}(\delta_0) \geq \hat{V}_{p,n}(\delta_\Delta) \text{ for all } t \in [0, \infty]. \tag{31}$$

Fix an $n$ and assume by induction that the statement holds for all $m > n$. First, suppose $k_n < \beta$ and $p \leq p_n^*$. Then, by (P), $\alpha_n(p) = 1$. So, $\hat{V}_{p,n}(\delta_0) = k_n$. This is the maximum payoff that can be achieved for all $m \geq n$, and thus (31) holds. Next, suppose $k_N \geq \beta$. In this case, by (P) and the inductive assumption, $\hat{V}_{p,n}(\delta_t) = \beta$ for all $t \in [0, \infty)$. Meanwhile, $\hat{V}_{p,n}(\delta_\infty) < \beta$. Thus, (31) holds. Finally, suppose $k_n < \beta$ or $p > p_n^*$. Then, for all $\Delta \in [0, \infty)$,

$$\hat{V}_{p,n}(\delta_\Delta) = (N - n) \int_0^\Delta k_{n+1} \alpha_{n+1}(p^i(s)) d\Psi^{-i}(s) + (1 - (N - n)\Psi^{-i}(\Delta)) \alpha_n(p(\Delta)),$$

where $\Psi^{-i}(\Delta) = \int_0^\infty (1 - F_{p,n}(s))^{N-n} e^{-\lambda(N-n)s} d((F_{p,n}(s) - 1)e^{-\lambda s})$. Differentiating, we have:

$$\frac{d}{d\Delta}\hat{V}_{p,n}(\delta_\Delta) = [k_{n+1}\alpha_{n+1}(p^i(\Delta)) - k_n \alpha_n(p(\Delta))] \Psi^{-i'}(\Delta)(N-n) + [1 - (N-n)\Psi^{-i}(\Delta)] p'(\Delta) \alpha_n'(p(\Delta)),$$

where $\Psi^{-i'}(\Delta) = \lambda(1 - F_{p,n}(\Delta))^{N-n} e^{-\lambda\Delta(N-n)} \frac{p(\Delta)}{\alpha_n(p(\Delta))}$. Substituting, we have

$$\frac{d}{d\Delta}\hat{V}_{p,n}(\delta_\Delta) = K[k_{n+1}\alpha_{n+1}(p^i(\Delta)) - V_{p^i(\Delta),n+1} - \beta(1-\alpha_n(p))(1-p)],$$

where $K > 0$ is a constant. Now, note that

$$V_{p^i(\Delta),n+1} \geq V_{p^i(\Delta),n+1}(\delta_0) = k_{n+1}\alpha_{n=1}(p^i(\Delta)) - (1-p(\Delta))(1-\alpha_n(p(\Delta))\beta.$$

Thus, $\frac{d}{d\Delta}\hat{V}_{p,n}(\delta_\Delta) \leq 0$, and therefore $\hat{V}_{p,n}(\delta_0) \geq \hat{V}_{p,n}(\delta_\Delta)$ for all $\Delta > 0$. It remains to show that $\hat{V}_{p,n}(\delta_0) > \hat{V}_{p,n}(\delta_\infty)$.

$$\hat{V}_{p,n}(\delta_\infty) = (N-n)\int_0^\infty k_n\alpha_{n+1}(p^i(s))d\Psi^{-i}(s) = \lim_{\Delta\to\infty}\hat{V}_{p,n}(\delta_\Delta) \leq \hat{V}_{p,n}(\delta_0).$$

□

**Proofs of Comparative Statics Results**

**Proof of Comparative Static 1.** First, we establish part (a). Fix all other parameters and let $0 < \beta < \tilde{\beta}$. Let $\alpha$ and $\tilde{\alpha}$ denote the equilibrium credibility functions under $\beta$ and $\tilde{\beta}$, respectively. Fix an $n$ and assume inductively that the proposition holds for $n+1$ if $n < N$. Note that for any $(p, n)$ and $t$, $p(t)$ will be the same under $\beta$ and $\tilde{\beta}$. Thus to show the above claim, it suffices to show that for any $p$, $\alpha_n(p)$ is weakly increasing in $\beta$, and strictly so whenever $\alpha_n(p) < 1$.

We begin by showing that $\alpha_n(p) = 1$ implies that $\tilde{\alpha}_n(p) = 1$. First, consider the case where $n = N$. By Proposition 2, $\alpha_N(p) = 1$ implies that $k_N \leq \beta$. Thus, $k_N < \tilde{\beta}$, which by Proposition 1 implies that $\tilde{\alpha}_N(p) = 1$. Next, consider the case where $n < N$, and assume $\alpha_n(p) = 1$. By Proposition 1, this implies that $k_n < \beta$ and $p \leq p_n^* \equiv \frac{\beta - k_n}{\beta - k_n/n}$. Further note that

$$\tilde{p}_n^* \equiv \frac{\tilde{\beta} - k_n}{\tilde{\beta} - k_n/n} > \frac{\beta - k_n}{\beta - k_n/n} \equiv p_n^*.$$

Thus, $k_n < \tilde{\beta}$ and $p < \tilde{p}_n^*$, which by Proposition 1 implies $\tilde{\alpha}_n(p) = 1$.

Now, suppose that $\alpha_n(p) < 1$. We wish to show that $\tilde{\alpha}_n(p) > \alpha_n(p)$. Suppose by contradiction that $\tilde{\alpha}_n(p) \leq \alpha_n(p)$. It follows from Proposition 2 that if $k_n > \tilde{\beta}$,

$$\lim_{q\to 0+}\alpha_n(q) = \beta/k_n < \tilde{\beta}/k_n = \lim_{q\to 0+}\tilde{\alpha}_n(q).$$

Meanwhile, if $k_n \leq \tilde{\beta}$. $\lim_{q\to\tilde{p}_n^*+}\alpha_n(q) < 1 = \lim_{q\to\tilde{p}_n^*+}\tilde{\alpha}_n(q)$. To see why the latter must hold, first consider the case where $n = N$. It follows from Lemma 5 that $\tilde{\alpha}_n(q) =$

1 for all $q$. Meanwhile, it follows again from Proposition 2 that $\alpha_N(q)$ is constant in $q$, and because $\alpha_N(p) < 1$, $\lim_{q\to\tilde{p}_n^*+}\alpha_N(q) < 1$. In the case where $n < N$, because $p_n^* < \tilde{p}_n^*$, it follows from Proposition 1 that $\alpha_n(\tilde{p}_n^*) < 1$.

Thus, we have that both when $k_n > \tilde{\beta}$ and when $k_n \leq \tilde{\beta}$, there exists some $\hat{p} < p$ such that $\tilde{\alpha}_n(\hat{p}) > \alpha_n(\hat{p})$ and $\tilde{\alpha}_n$, $\alpha_n$ satisfy (ODE) on $[\hat{p}, p]$ , for their respective value of $\beta$. Thus, there exists a $q \in [\hat{p}, p]$ such that $\alpha_n(q) = \tilde{\alpha}_n(q)$ and $\alpha_n'(q) \geq \tilde{\alpha}_n'(q)$. It follows from (ODE) that in order for the above two conditions to hold, it must be that

$$X \equiv (\beta - \tilde{\beta})(\frac{1-\alpha_n(q)}{\alpha_n(q)})(1-q) + \frac{V_{q^i,n+1} - \tilde{V}_{q^i,n+1}}{\alpha_n(q)} \geq 0. \tag{32}$$

where $V$ and $\tilde{V}$ denote the value functions under $\beta$ and $\tilde{\beta}$, respectively. First consider the case where $n = N$. Then $V_{q^i,n+1} = V_{\tilde{q}^i,n+1} = 0$, and thus $X < 0$, contradicting (32).

Next, consider the case where $n < N$. First suppose that $\alpha_{n+1}(q^i) = 1$. It follows from the inductive assumption that $\tilde{\alpha}_{n+1}(q^i) = 1$. Thus, by Lemma 5, $V_{q^i,n+1} = \frac{k_{n+1}q^i}{N-n} = \tilde{V}_{q^i,n+1}$. Again this implies that $X < 0$, contradicting (32). Now, suppose that $\alpha_{n+1}(q^i) < 1$. It then follows from Lemma 2 that $V_{q^i,n+1} = k_{n+1}\alpha_{n+1}(q^i) - \beta(1-q^i)$. Furthermore,

$$\tilde{V}_{q^i,n+1} = \tilde{V}_{q^i,n+1}(\delta_0) = k_{n+1}\tilde{\alpha}_{n+1}(q^i) - \tilde{\beta}(1-q^i).$$

Thus, recalling that $q^i = \alpha_{n+1}(q) + (1-\alpha_{n+1}(q))q$, we have

$$V_{q^i,n+1} - \tilde{V}_{q^i,n+1} \leq k_{n+1}(\alpha_{n+1}(q^i) - \tilde{\alpha}_{n+1}(q^i)).$$

Substituting this into the above expression for $X$, we obtain

$$X \leq \frac{k_{n+1}(\alpha_{n+1}(q^i) - \tilde{\alpha}_{n+1}(q^i))}{\alpha_n(q)} < 0.$$

where the strict inequality follows from the inductive assumption that $\alpha_{n+1}(q^i) < \tilde{\alpha}_{n+1}(q^i))$. Again, this is a contradiction of (32). $\square$

**Proof of Corollary 4.** Fix all other parameters and let $0 < \beta < \tilde{\beta}$. Assume a winner-takes-all setting ($k_n = 0$ for $n > 1$). Let $\alpha$ ($V$) and $\tilde{\alpha}$ ($\tilde{V}$) denote the equilibrium credibility (value function) under $\beta$ and $\tilde{\beta}$, respectively. We want to show that $V_{p_0,1} \leq \tilde{V}_{p_0,1}$ and $V_{p_0,1} < \tilde{V}_{p_0,1}$ when $\alpha_1(p_0) < 1$.

First, suppose $\alpha_1(p_0) = 1$. Because firms are truthful in this case,

$$V_{p_0,1} = V_{p_0,1}(\delta_\infty) = \frac{k_1}{N},$$

where the exact same equality holds under $\tilde{\beta}$. Thus, $V_{p_0,1} = \tilde{V}_{p_0,1}$.

Next, suppose $\alpha_1(p_0) < 1$. It follows from Lemma 2 that $V_{p_0,1} = V_{p_0,1}(\delta_\infty)$ and $\tilde{V}_{p_0,1} = \tilde{V}_{p_0,1}(\delta_\infty)$ . Now, note that

$$V_{p_0,1}(\delta_\infty) = \int_0^\infty k_1\alpha_1(p_0(s))\psi^i(s)ds \text{ and } \tilde{V}_{p_0,1}(\delta_\infty) = \int_0^\infty k_1\tilde{\alpha}_1(p_0(s))\tilde{\psi}^i(s)ds, \text{ where}$$

$\psi^i(s) = p\lambda e^{-\lambda sN}(1 - F_{p_0,1}(s))^{N-1}$ and $\tilde{\psi}^i(s) = p\lambda e^{-\lambda sN}(1 - \tilde{F}_{p_0,1}(s))^{N-1}$, and $F$ ($\tilde{F}$) is the equilibrium strategy under $\beta$ ($\tilde{\beta}$). Now, note by Comparative Static 1 that

$$\alpha_1(p_0(s)) \leq \tilde{\alpha}_1(p_0(s)), \tag{33}$$

where the in equality holds strictly for some interval of $s$. Likewise,

$$b_1(p_0(s)) \geq \tilde{b}_1(p_0(s)),$$

where the in equality holds strictly for some interval of $s$. This implies

$$F_1(p_0(s)) > \tilde{F}_1(p_0(s)), \text{ for all } s > 0.$$

This, combined with (33), implies that $V_{p_0,1} < \tilde{V}_{p_0,1}$.

□

**Proof of Comparative Static 2.** Let $\tilde{\lambda} > \lambda > 0$, and let $\alpha$, $\tilde{\alpha}$ denote the equilibria under $\lambda$ and $\tilde{\lambda}$, respectively, fixing all other parameters. We begin by showing that $\tilde{\alpha}_n(p) = \alpha_n(p)$ for any $p$ and $n$. Fix an $n$ and assume inductively that if $n < N$, $\alpha_{n+1}(p) = \tilde{\alpha}_{n+1}(p)$ for all $p$ on-path. Let $V$, $\tilde{V}$ denote the value functions under the equilibria associated with $\lambda$ and $\tilde{\lambda}$, respectively. Note that $V_{p,n+1} = \tilde{V}_{p,n+1}$ for all $p$ on-path. In the case where $n = N$, $V_{p,n+1} = \tilde{V}_{p,n+1} = 0$, and thus this holds trivially. In the case where $n < N$, this follows from the inductive assumption.

By Lemma 5, $\alpha_n$ and $\tilde{\alpha}_n$ must both be a solution to (P) at all $(p, n)$ on-path, which does not depend on $\lambda$. By Theorem 1, the solution to (P) is unique, and thus $\alpha_n(p) = \tilde{\alpha}_n(p)$ at all $(p, n)$ on-path. Now fixing any $p$ and $n$, let $p(t)$ and $\tilde{p}(t)$ denote the common beliefs under $\lambda$ and $\tilde{\lambda}$, respectively. It follows from (2) that $p(t) > \tilde{p}(t)$ for all $t > 0$. Thus, because $\alpha_n(p)$ and $\tilde{\alpha}_n(p)$ are both weakly decreasing in $p$ (Proposition 3), it follows that $\alpha_n(p(t)) \leq \tilde{\alpha}_n(p(t))$. Furthermore, since $\tilde{\alpha}(p)$ is strictly decreasing in $p$ (Proposition 3) whenever $\alpha_n(p) < 1$ and $k_N > \beta$, it follows that $\alpha_n(p(t)) < \alpha_n(\tilde{p(t)})$.

□

**Proof of Comparative Static 3.** Let $\alpha$ and $\tilde{\alpha}$ denote the equilibria under $N$ and $N+1$ firms, respectively, fixing all other parameters. We begin by showing that for all $p$, $\alpha_n(p) \geq \tilde{\alpha}_n(p)$, and $\alpha_n(p) > \tilde{\alpha}_n(p)$ when $\alpha_n(p) < 1$. To this end, fix an $n \in \{1, ..., N\}$ and assume inductively that the claim holds for $n+1$ whenever $n < N$. We begin by showing that $\tilde{\alpha}_n(p) = 1$ implies that $\alpha_n(p) = 1$. Suppose that $\tilde{\alpha}_n(p) = 1$. By Proposition 1, $\beta > k_n$ and $p < \tilde{p}_n^* \equiv \frac{\beta - k_n}{\beta - k_n/(N+1-n)}$. Because $p_n^* \equiv \frac{\beta - k_n}{\beta - k_n/(N-n)} > \tilde{p}_n^*$, it follows from Proposition 1 that $\alpha_n(p) = 1$.

Now consider the case where $\tilde{\alpha}_n(p) < 1$. We wish to show that $\tilde{\alpha}_n(p) < \alpha_n(p)$. To this end, we begin by making the following observation:

$$\text{If } \alpha_n \text{ and } \tilde{\alpha}_n \text{ both satisfy (ODE) at } q, \text{ and } \alpha_n(q) = \tilde{\alpha}_n(q), \text{ then } \alpha_n'(q) > \tilde{\alpha}_n'(q). \tag{34}$$

Let us now establish this. Note first that for $\alpha_n$ and $\tilde{\alpha}_n$ to both satisfy (ODE) at $q$, given that $\alpha_n(q) = \tilde{\alpha}_n(q)$, the following must hold:

$$\alpha_n'(q) = \frac{-1}{k_n(1-q)\alpha_n(q)} \frac{N-n}{N-n+1} (k_n \alpha_n(q) - V_{q^i,n+1} - \beta(1-\alpha_n(q))(1-q))$$

$$\tilde{\alpha}_n'(q) = \frac{-1}{k_n(1-q)\alpha_n(q)} \frac{N-n+1}{N-n+2} (k_n \alpha_n(q) - \tilde{V}_{q^i,n+1} - \beta(1-\alpha_n(q))(1-q)),$$

where $V$ and $\tilde{V}$ denote the value functions under the equilibria with $N$ and $N+1$ total firms, respectively. Note that if $n = N$, $\alpha_n'(q) = 0$. Meanwhile, by Proposition 3, $\tilde{\alpha}_n'(q) < 0$. Thus, $\tilde{\alpha}_n'(q) < \alpha_n(q)$ must hold. Next, consider the case where $n < N$. We begin by observing that $V_{q^i,n+1} > \tilde{V}_{q^i,n+1}$. To see why this must hold, first consider the case where $\tilde{\alpha}_{n+1}(q^i) = 1$. It then follows from the inductive assumption that $\alpha_n(q^i) = 1$. Then, by Lemma 5,

$$\tilde{V}_{q^i,n+1} = \tilde{V}_{q^i,n+1}(\delta_\infty) = \frac{k_{n+1}q^i}{N-n} < \frac{k_{n+1}q^i}{N-n-1} = V_{q^i,n+1}(\delta_\infty) = V_{q^i,n+1}.$$

Next, consider the case where $\tilde{\alpha}_n(q^i) < 1$. In this case, it follows from Lemma 2 that

$$\begin{aligned} \tilde{V}_{q^i,n+1} = \tilde{V}_{q^i,n+1}(\delta_0) = k_{n+1}\tilde{\alpha}_{n+1}(q^i) - \beta(1-q^i) &< k_{n+1}\alpha_{n+1}(q^i) - \beta(1-q^i) \\ &= V_{q^i,n+1}(\delta_0) \leq V_{q^i,n+1}, \end{aligned}$$

where the strict inequality follows from the inductive assumption made above. Examining the two ODEs listed above, since by Proposition 3, $\alpha_n'(q) \leq 0$, it follows that $\tilde{\alpha}_n'(q) < \alpha_n'(q)$.

Now, assume by contradiction that $\alpha_n(p) \leq \tilde{\alpha}_n(p)$. We begin by showing that there exists a $q^* < p$ such that $\tilde{\alpha}_n(q^*) < \alpha_n(q^*)$. First consider the case where $k_n \geq \beta$. Then, by Proposition 2,

$$\lim_{q \to 0+} \alpha_n(q) = \lim_{q \to 0+} \tilde{\alpha}_n(q) = \frac{\beta}{k_n}.$$

Then, by the continuous differentiability of $\alpha_n$ and $\tilde{\alpha}_n$ on $(0, p)$, it follows from Equation 34 that for some $q^* < p$ sufficiently small $\alpha_n(q^*) > \tilde{\alpha}_n(q^*)$. Next, consider the case where $k_n < \beta$, and let $p_n^* \equiv \frac{\beta - k_n}{\beta/(N-n+1) - k_n}$. Note by Proposition 1 that $\alpha_n(p_n^*) = 1$. Meanwhile, because $p_n^* < \tilde{p}_n^* \equiv \frac{\beta - k_n}{\beta/(N-n+2) - k_n}$, it follows from Proposition 1 that $\tilde{\alpha}_n(p_n^*) < 1$, and thus, we have for $q^* = p_n^*$, $\tilde{\alpha}_n(q^*) < \alpha_n(q^*)$.

Since $\tilde{\alpha}_n(q^*) < \alpha_n(q^*)$ and $\tilde{\alpha}_n(p) \geq \alpha_n(p)$, by the continuous differentiability of $\alpha$ on $[q^*, p]$, there must exist some $q \in (q^*, p]$ such that $\alpha_n(q) = \tilde{\alpha}_n(q)$ and $\alpha_n'(q) \leq \tilde{\alpha}_n'(q)$. However, this is a contradiction of (34).

Now fixing any $p$ and $n$, let $p(t)$ and $\tilde{p}(t)$ denote the common beliefs under $N$ and $N+1$ firms, respectively. We wish to show that on some interval $[0, \bar{t}]$, where $\bar{t} > 0$, $\alpha_n(p(t)) \geq \tilde{\alpha}_n(\tilde{p}(t))$ is weakly increasing in $t$, and strictly so whenever $\alpha_n(p(t)) < 1$. First consider the case where $\alpha_n(p(t)) = 1$. In this case, the statement holds trivially. Next, consider the case where $\alpha_n(p) < 1$. It follows from the above that $\alpha_n(p) > \tilde{\alpha}_n(p)$. Now note that it follows from (2) that $\lim_{t \to 0+} p(t) - \tilde{p}(t) = 0$. Since $\alpha_n(p(t))$ and $\tilde{\alpha}_n(\tilde{p}(t))$ are both continuous in $t$ (Lemma 4), it follows that for some $\bar{t} > 0$, $\alpha_n(p(t)) > \tilde{\alpha}_n(\tilde{p}(t))$ for all $t \in [0, \bar{t}]$. ☐

### Microfoundation for market share

In the main text, I assume that the firm's market share from reporting a story is $k_n \alpha$. Here I provide a microfoundation for this.

Let $\mathcal{N} \equiv \{1, ..., N\}$ denote the set of news firms. Suppose there is a mass $K > 0$ of consumers, who are indexed by $x$. Each consumer $x$ *subscribes* to some subset $S_x$ of the firms. I.e., for all $x$, $S_x \subseteq \mathcal{N}$. Let $S_x$ denote consumer $x$'s *subscription set*. Fixing any $S \subseteq \{1, ..., N\}$, let $m(S)$ denote the mass of consumers $x$ such that $S_x = S$, where $\sum_{S \in 2^{\mathcal{N}}} m(S) = K$. Assume that the mass of consumers with a given subscription set does not depend on the identity of the firms within that set, but only on the number of firms in the set. Formally, suppose that there exists $\gamma_0, \gamma_1, ..., \gamma_N \geq 0$ such that

$$m(S) = \gamma_n \text{ if and only if } |S| = n, \text{ where } \sum_{n=1}^{N} \gamma_n \binom{N}{n} = K.$$

Define $i$'s *market share* to be the mass of consumers who read the story. We assume that a consumer reads a story if she both *considers* the story, and finds it optimal to read it. To formalize this, let $\hat{S} \subseteq \mathcal{N}$ denote the set of firms who reported before $i$. A consumer $x$ will *consider* a story if and only if:

1. The firm is in the consumer's subscription set, i.e., $i \in S_x$.

2. The consumer has not previously considered the story. I.e., $j \notin S_x$ for all $j \in \hat{S}$.

The mass of consumers who consider firm $i$'s story is then given by

$$\sum_{j=1}^{N-n} \binom{N-n}{j} \gamma_{j+1} \equiv k_n,$$

where $n$ is the order of $i$'s report. Next, suppose consumer $x$ faces a cost $c_x$ of reading a story. Suppose that $c_x$ is i.i.d. across $x$, that for any $x$, $c_x$ is uniformly distributed on $[0,1]$, and that $c_x$ is independent of $x$'s consideration set. Then $x$'s payoff from reading a story is $\mathbb{I}[\theta = 1] - c_x$. That is, the consumer will incur a cost $c_x$ from reading the story, and a benefit of $1$ only if the story is true. Meanwhile, the consumer's payoff from not reading a story is $\mathbb{I}[\theta = 0]$. Namely, the consumer enjoys a payoff of $1$ from refusing to story that is untrue. Assuming consumers maximize expected utility, $x$ will read the story if and only if

$$\alpha + (1-\alpha)p - c_x \geq (1-\alpha)(1-p) \Leftrightarrow c_i \leq \alpha$$

where $\alpha$ is the credibility of $i$'s story. Thus i's market share is $k_n \alpha$.

**Equilibrium credibility**

Here, I justify equation (4) by showing that it is the limit of Bayes-consistent beliefs under a discrete approximation of the game presented in Section 2. To this end, for any $\varepsilon > 0$, let the *$\varepsilon$-approximation* of the game be identical to the game presented in section (2), except with the following modification: any report made by a firm on $[0, \varepsilon]$ is observed by all other players (including the consumer) at $\varepsilon$. That is, rather than observing $t_i$, the players observe $\tilde{t}_i$, where $\tilde{t}_i \equiv max\{t_i, \varepsilon\}$

At any $(p, n)$ that is on-path, let $\alpha_n^\varepsilon(p)$ denote the firm's credibility, i.e., the consumer's belief that $s_i \leq \varepsilon$ given that $\tilde{t}_i = \varepsilon$, under the $\varepsilon$-approximation of the game. Let $\alpha_n$ denote the right-limit of the $\alpha_n^\varepsilon$. Then: $\alpha_n(p) \equiv \lim_{\varepsilon \to 0+} \alpha_n^\varepsilon(p)$ I now establish that $\alpha_n(p)$ is given by (4) at any $(p, n)$ on-path.

**Claim 3.** *For any $(p, n)$ on-path,*

$$\alpha_n(p) = \begin{cases} \frac{\lambda p}{\lambda p + b_n(p)} & \text{if } F_{p,n}(0) = 0 \\ 0 & \text{if } F_{p,n}(0) > 0 \end{cases}$$

**Proof.** For any $\varepsilon > 0$, it follows from Bayes Rule that

$$\alpha_n^{\varepsilon}(p) = \frac{p(1 - e^{-\lambda\varepsilon})}{p(1 - e^{-\lambda\varepsilon}) + F_{p,n}(\varepsilon)e^{-\lambda\varepsilon}}.$$

If $F_{p,n}(0) = 0$, it follows from L'Hôpital's Rule that:

$$\lim_{\varepsilon \to 0+} \alpha_n^{\varepsilon}(p) = \frac{\lambda p}{\lambda p + b_n(p)}.$$

If $F_{p,n}(0) > 0$, it follows from the right-continuity of $F_{p,n}$ that

$$\lim_{\varepsilon \to 0+} \alpha_n^{\varepsilon}(p) = \frac{0}{0 + \lim_{\varepsilon \to 0+} F_{p,n}(\varepsilon)} = 0.$$

□

### Extension: heterogeneous ability

I now consider an extension in which firms have heterogeneous learning abilities. This will shed light on how a firm's credibility correlates with its ability in equilibrium.

The extended model is identical to the model above except for three changes. First, rather than assuming that each firm is endowed with the same ability $\lambda$, each firm $i$ is endowed with an firm-specific ability $\lambda^i$, which is common knowledge. Second, for tractability, I restrict attention to a winner-takes-all setting: i.e., I assume $k_n = 0$ for all $n > 1$. Finally, I relax the equilibrium symmetry assumption. Accordingly, let $\alpha^i$ denote the credibility of firm $i$.

I obtain an intuitive result: firms with higher ability are more credible in equilibrium.

**Proposition 5.** *For all $(i, j)$ such that $\lambda^i < \lambda^j$, $\alpha_1^i(p(t)) \leq \alpha_1^j(p(t))$. Furthermore, this inequality is strict whenever $\alpha_1^i(p(t)) < 1$.*

Proposition 5 states that regardless of when a report is made, a firm with higher ability is weakly more credible, and strictly so whenever firms are not fully truthful. Let us consider why this correlation arises. First, note that high ability firms are able to confirm a story more quickly and thus, all else equal, pose a greater preemptive threat in equilibrium. This in turn implies that in comparison to a high-ability firm, a low-ability firm faces a greater preemptive threat. Thus, the low-ability firm finds immediate faking more advantageous. In light of this, the firms' credibilities must adjust in such a way to preserve their respective indifference conditions. This is achieved endogenously by means of a lower credibility for the low-ability firm, which ensures that it has less to gain from faking.

### Proofs: heterogeneous learning ability

Here, we consider the extended model presented in above. The objective is to establish Proposition 5. This proof will require extending certain results established in the baseline model to the extended model. Regarding Lemmas 1-4, I will take for granted that these hold under the extended model. Formal proofs of this are omitted as all proofs presented under the baseline model will also apply to the extended setting.

Next, I establish that Proposition 1 holds under the extended model. This claim is presented as Proposition 1′. In the analysis below, I let $V^i_{p,n}$ denote firm $i$'s value.

**Proposition 1′.** *For all $i$, there exists a $p^{i*} \in (0,1]$ such that at any $p$ on-path, $\alpha^i_1(p) = 1$ if and only if the following two conditions hold:*

1. $k_1 \leq \beta$
2. $p \leq p^{i*}$

*Furthermore, $p^{j*} > p^{i*}$ whenever $\lambda^j > \lambda^i$.*

**Proof.** Fix an $i$. Suppose that $k_1 \leq \beta$. By identical reasoning as Proposition 1, for all $q < \frac{\beta - k_1}{k_1}$, $\alpha^i_1(q) = 1$. Let

$$p^{i*} \equiv sup\{p|\alpha^i_1(p) = 1 \text{ for all } q < p\}.$$

It follows by definition that $\alpha^i_1(p) = 1$ for all $p \leq p^{i*}_1$.

Next, we will show that $\alpha^i_1(q) < 1$ whenever $k_1 > \beta$ or $p > p^{i*}_1$. Suppose not by contradiction. First, consider the case where $k_1 > \beta$ and $\alpha^i_1(p) = 1$ for some $p$. Then we have that

$$V^i_{p,1}(\delta_0) = k_1 p + (k_1 - \beta)(1-p) > k_1 p \geq V^i_{p,1}(\delta_\infty)$$

Thus, $i$ can profitably deviate at $p$. Contradiction. Next, consider the case where $q > p^{i*}_1$ and $\alpha^i_1(p) = 1$. In this case, a contradiction follows from identical reasoning to what is presented in Proposition 1.

Finally, we show that $p^{j*} > p^{i*}$ whenever $\lambda^j > \lambda^i$. Suppose by contradiction that $p^{j*} \leq p^{i*}$. Note that because $j$ is truth telling at $(p^{j*}, n = 1)$, $V^j_{p^{j*},1}(\delta_\infty) \geq V^j_{p^{j*},1}(\delta_0)$. Furthermore, because $p^{j*} \leq p^{i*}$, $i$ is also truthful at $(p_n^{j*}, n = 1)$. Thus,

$$V^j_{p_1^{j*},1}(\delta_0) = V^i_{p_1^{j*},1}(\delta_\infty) = k_1 - \beta(1-p).$$

Now, note that because $\lambda^j > \lambda^i$,

$$V^j_{p_1^{j*},1}(\delta_\infty) > V^i_{p_1^{j*},1}(\delta_\infty).$$

Combining these inequalities we have $V^i_{p_1^{j*},1}(\delta_\infty) < V^i_{p_1^{j*},1}(\delta_0)$. However, because $\alpha_1^i(p^{j*}) = 1$, $V^j_{p_n^{j*},1} = V^j_{p_n^{j*},1}(\delta_\infty)$. Contradiction. □

Next, we extend Proposition 2 to this setting. Note this entails deriving an ODE that applies to this extended model, (ODE-$i$).

**Proposition 2'.** *In equilibrium, for any $p$ on-path, if $k_1 \geq \beta$ or $p > p^{i*}$, then the following must be satisfied:*

$$\alpha_1^{i\prime}(p) = -\frac{\beta}{k_1}\left(\frac{\sum_{j\neq i}\lambda^j}{\sum_j \lambda^j}\right) - \frac{\sum_{j\neq i}\frac{\lambda^j}{\alpha_1^j(p)}}{\sum_j \lambda^j(1-p)}\left[\alpha^i(p) - \frac{\beta}{k_1}(1-p)\right]. \qquad \text{(ODE-}i\text{)}$$

*In addition, $\lim_{p\to 0+}\alpha_1^i(p) = \beta/k_1$ must hold if $k_1 > \beta$, and $\lim_{p\to p^{i*}+}\alpha_1^i(p) = 1$ if $k_1 \leq \beta$.*

**Proof.** Let us first establish that (ODE-$i$) must hold under the conditions specified.

When $k_1 \geq \beta$ or $p > p^{i*}$, it follows from Proposition 1' that $\alpha_1^i(p(t)) < 1$. It then follows from Lemma 2 that there exists an $\varepsilon > 0$ such that for all $\Delta \in (0, \varepsilon)$,

$$\frac{V^i_{p,1}(\delta_\Delta) - V^i_{p,1}(\delta_0)}{\Delta} = 0.$$

Recall that $V^i_{p,1}(\delta_0) = k_1\alpha_1^i(p) - \beta(1-p)$. Meanwhile,

$$V^i_{p,1}(\delta_\Delta) = \int_0^\Delta k_1\alpha_1^i(p(s))\Psi^i(s)ds + (1 - \sum_j \lim_{s\to\Delta-}\Psi^j(s))[k_1\alpha_1(p(\Delta)) - \beta(1-p(\Delta))],$$

where $\Psi$ is the first-report distribution associated with the strategy profile in which

$i$ plays $\delta_\infty$ and all $j \neq i$ play the equilibrium strategy $F^j_{p,1}$. Specifically, for all $s > 0$,

$$\Psi^i(s) = p\lambda^i \int_0^s e^{-\sum_j \lambda^j r} \prod_{j \neq i} (1 - F^i_{p,1}(r))dr$$

and for $j \neq i$,

$$\Psi^j(s) = p \int_0^s e^{-\sum_{k \neq j} \lambda^k r} \prod_{k \neq i \neq j} (1 - F^k_{p,1}(r))d(-e^{-\lambda^j r}(1 - F^j_{p,1}(r)))$$
$$+(1-p) \int_0^s \prod_{k \neq i \neq j} (1 - F^k_{p,1}(r))dF^j_{p,1}(r).$$

Substituting these two expressions into the above equation for $V^i_{p,1}(\delta_\Delta)$ and following the same sequence of steps as in the proof of Proposition 2 yields (ODE-$i$).

The two limit conditions are established by the same reasoning presented in the proof of Proposition 2. □

**Proof of Proposition 5.** Fix any $(i, j)$ such that $\lambda^i > \lambda^j$. We want to show that $\alpha^i_1(p(t)) \leq \alpha^j_1(p(t))$ and that $\alpha^i_1(p(t)) < \alpha^j_1(p(t))$ whenever $\alpha^i_1(p(t)) < 1$. First suppose $\alpha^i_1(p) = 1$. In this case, $\alpha^i_1(p) \geq \alpha^j_1(p)$ is trivially satisfied.

Next, suppose $\alpha^i_1(p) < 1$. We want to show that $\alpha^i_1(p) > \alpha^j_1(p)$. Suppose by contradiction that $\alpha^i_1(p) \leq \alpha^j_1(p)$. First consider the case where $k_1 < \beta$. Then, let

$$q^* \equiv \inf\{q | \alpha^j_1(p) < 1 \text{ and } \alpha^j_1(p) < \alpha^i_1(p)\}.$$

Because the $\alpha^i_1$ are continuous, it follows from Proposition 1′, and the assumption that $\alpha^i_1(p) \leq \alpha^j_1(p)$, that $q^* < p$ exists. Again, by continuity, $\alpha^j_1(q^*) = \alpha^i_1(q^*)$. It then follows from (ODE-$i$) that $\alpha^{j\prime}_1(q^*) < \alpha^{i\prime}_1(q^*)$. But this implies that for some $q > q^*$, $\alpha^j_1(q^*) > \alpha^i_1(q^*)$. Contradiction.

Next, consider the case where $k_1 \geq \beta$. Recall by Proposition 2′ that $\lim_{p\to 0+} \alpha^i_1(p) = \lim_{p\to 0+} \alpha^j_1(p)$. Thus, there exists some $q \in (0, p]$ such that $\alpha^i_1(p) \leq \alpha^j_1(p)$ and $\alpha^{i\prime}_1(p) \leq \alpha^{j\prime}_1(p)$. However, it again follows from (ODE-$i$) that $\alpha^{i\prime}_1(p) > \alpha^{j\prime}_1(p)$. Contradiction. □