Competition and Herding in Breaking News

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Abstract

I present a dynamic model of breaking news. Firms are rewarded for preempting their competitors and for making credible reports. Errors occur when firms fake, reporting without evidence. In equilibrium, competition and observational learning exacerbate errors and give rise to rich dynamics in reporting. Errors propagate through the market via a *copycat effect*, where a new report triggers a surge in faking. The copycat effect is driven by observational learning but depends on the nature of competition, as it is more prevalent in settings where there are smaller benefits to being a first reporter. This behavior implies not only the propagation of errors but also herding on the timing of news.

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1. Introduction

What a newspaper needs in its news, in its headlines, and on its editorial page is terseness, humor, descriptive power, [...] and accuracy, accuracy, accuracy!

— Joseph Pulitzer

Accuracy is often considered the core tenet of news media. This belief is widely held by consumers: the majority of US survey respondents listed accuracy as a primary function of news, valuing it over thorough coverage, unbiasedness, and relevance (Pew Research Center, 2019).

Despite this, news firms often make factual errors during major breaking news events. In the aftermath of the 9/11 attacks, cable news stations made several statements that were false: NBC News reported an explosion outside the Pentagon, CNN reported a fire outside the national mall, and CBS News claimed the existence of a car bomb outside the state department.[1] Erroneous reporting has been endemic to terrorist attacks in general, with news media misidentifying perpetrators or other key details of the Boston bombings, Sandy Hook massacre, and London bombings. Errors have surrounded political scandals as well. In 2004, CBS News published the Killian Documents, a collection of memos which called into question George W. Bush's military record. These documents could never be authenticated and were widely believed to be forged. In 2017, ABC News falsely reported that Michael Flynn would testify that Donald Trump had directed him to make contact with the Russians.[2]

Errors can also propagate through the market for news. Following the Boston bombings, CNN reported that a suspect was in custody when this was not true, and was shortly followed by Fox News, the Boston Globe and other news firms in doing so.[3] During the vote tallying of the 2000 US presidential election, several news stations called Florida in favor of Al Gore in close succession, and within three hours began withdrawing this call.

Even though errors are widespread, they are costly to news firms as they can be reputationally damaging. This was especially true of the *Rolling Stone* scandal,

[1] https://www.reuters.com/article/idUS182595581320121217

[2] https://www.nytimes.com/2017/12/02/us/brian-ross-suspended-abc.html

[3] https://www.nytimes.com/2013/04/18/business/media/fbi-criticizes-false-reports-of-a-bombing-arrest.html

in which the magazine falsely accused a group of University of Virginia students of sexual assault. Not only was the journalistic failure widely reported, it also resulted in several publicized lawsuits against the magazine. Furthermore, errors can lead firms to oust journalists in an apparent effort to protect their reputations. This was evident in the terminations of Dan Rather and Brian Ross—both lead journalists at major news stations—following their respective reporting blunders.

The objective of this paper is twofold. First, I seek to understand why reporting errors are pervasive despite their costliness to firms. In particular, I consider how the strategic and learning environment news firms face can induce them to commit errors that are avoidable. My second objective is to study the dynamics of breaking news. Namely, I ask how reporting behavior changes over the course of a news cycle and how firms react to a report by a rival firm.

To this end, I present a dynamic model of breaking news. Firms learn privately about a story by receiving confirmation that it is true, and choose if and when to report it. Errors occur when firms fake, i.e., report despite lacking confirmation. Because reports are public, firms also learn by observing the reporting behavior of opponents. Regarding payoffs, firms are penalized for errors but rewarded for market share, which depends on two qualities of the report. First, all else equal, a firm that preempts its rivals enjoys greater market share. Second, market share depends on the credibility of the firm's report, which is the consumers' belief that the story was confirmed to be true. That is, a report is consumed only to the extent that there is trust in the firm's journalistic standards.

I establish existence and uniqueness of an equilibrium. Under this equilibrium, the firm randomizes across faking times: fake reports are made as if they are being generated by a non-homogeneous Poisson process. The indifference condition that supports this mixing implies an ordinary differential equation (ODE) on the arrival rate of fake reports, and the equilibrium is characterized by a recursive system of these ODEs, a fact that is central to the analysis.

In equilibrium, competition and observational learning exacerbate errors and give rise to rich dynamics in news reporting, including herding. Furthermore, the way in which these two features of the model interact determines how susceptible firms are to herding and thus propagating errors. Competition engenders a preemptive motive, which makes faking more valuable. Thus, credibility falls in equilibrium to restore indifference, giving rise to more errors. Competition also yields dynamics that are otherwise absent: whenever there is a preemptive motive, firms become gradually more credible over time in the absence of new reports. In other words, quick reports are less trustworthy than those made with greater delay. The reason for this gradual improvement in credibility lies in the firms' incentives: the risk of preemption introduces an endogenous cost to delay, and the firm must somehow be compensated for this cost to ensure that its indifference condition is satisfied. This is achieved by means of increasing credibility. That is, credibility increases to mitigate the haste-inducing effects of preemption. Notably, this preemptive motive is not just an artifact of competition, but rather an equilibrium phenomenon: firms have an incentive to preempt each other when the cost of error is large, but when this cost is sufficiently small, preemptive motives are nullified by the higher credibility of later reports.

Meanwhile, new reports trigger instantaneous and persistent changes in reporting behavior. This can take the form of a *copycat effect*, in which one firm's report triggers an increase in faking by other firms. The copycat effect exacerbates errors by causing existing errors to propagate through the market. This behavior is consistent not only with herding on the decision to report a story, but also herding on the timing of news reports, both correct and erroneous. In addition to anecdotal evidence of clustering in the timing of news errors (including the aforementioned reporting surrounding the Boston bombings and 2000 US presidential election), such herding has been empirically documented. Cagé, Hervé, and Viaud (2020) find that in 25% of cases, a news story is reported by a different media outlet within 4 minutes of being published by the original news breaker. I provide a rationale for such herding that is grounded in the strategic and learning environment news media face.

More precisely, I show that the copycat effect is driven by observational learning, but the nature of competition determines whether this phenomenon occurs and how pronounced it is. Observational learning drives this behavior because a report by one firm, even if it is erroneous, makes other firms more confident that the story is true. This then implies a lower perceived risk of error and a greater threat of preemption, both of which yield the other firms more inclined to fake. However, the nature of competition is crucial, as the copycat effect is both more likely to occur and more pronounced in settings where the benefits to being an early (e.g., first) reporter are relatively small. This is because smaller preemptive motives curb the probability with which errors are originated, so existing reports are viewed more credibly by other firms, making the other firms more likely to spread errors. Thus, reducing preemptive motives indirectly increases the incidence of errors by enhancing their propagation through the market. Meanwhile, when there are significant benefits to being an early reporter, *anti-herding* arises, a converse phenomenon in which a new report triggers a decline in faking. This suggests that competition has a nuanced effect on news reporting: while preemptive motives can lead early reporters to commit errors, they can also limit the propagation of errors to other news organizations. This also gives rise to the testable implication that herding in news is more pronounced in settings with less overlap in firms' consumer bases, i.e., where consumers multi-home less across news outlets.

Finally, I study the welfare implications of this equilibrium and conduct comparative statics. I find that competition reduces consumer welfare both when there are large gains to being the first reporter and when firms face little preemptive motive. When the gains to being the first reporter are large, competition deteriorates welfare by reducing the credibility of early reports. Meanwhile, when preemptive motives are small, welfare declines instead because of the copycat effect, which increases the probability that later reports are fake. This illustrates that observational learning can deteriorate consumer welfare even in environments where firms face little preemptive motive. With regards to comparative statics, I consider how the equilibrium responds to changes in the cost of error, the speed of learning, and market entry. A higher cost of error improves credibility by making faking less profitable, while faster learning curbs faking by causing firms to become pessimistic about the story more quickly. Firm entry has a more nuanced effect: it deteriorates credibility early on in the news cycle by exacerbating preemptive motives, but this is mitigated later on as the entering firm improves the market's ability to learn observationally that the story is false.

Related Literature This paper contributes to an extensive literature on competition in news, as surveyed by Gentzkow and Shapiro (2008). I contribute to the subset of this literature that considers how competition affects news accuracy. In Chen and Suen (2023) competition reduces accuracy by diluting consumer attention, while in Galperti and Trevino (2020) a strong consumer motive to coordinate actions can sometimes lead competitive news firms to supply lower-accuracy news. In contrast, I consider a setting where news firms' core tradeoff is between accuracy and the strategic cost of being preempted. This is studied in papers on preemption in news (Lin (2014), Pant and Trombetta (2025), Andreottola and De Moragas (2025)). Pant and Trombetta (2025) model news firms who face a tradeoff between a preemptive and reputation-building motives, and find that competition can improve information quality when preemptive motives are limited. As in the model I present, in Lin (2014) firms dynamically learn about a story and decide when to report it. I build on this literature by incorporating two key elements of breaking news: credibility and observational learning. Together, these two features drive equilibrium behavior, including herding. To my knowledge, this is the first paper to consider how observational learning by news firms affects accuracy and how competition can drive dynamics in news reporting.

This paper also contributes to the literature on observational learning with endogenously-timed decisions. Herding on actions as well as decision timing is well-documented (Chamley and Gale (1994), Grenadier (1999), Gul and Lundholm (1995), Murto and Välimäki (2011)). Chamley and Gale (1994) and Grenadier (1999) study investment timing games and show that endogenously-timed information cascades can occur, where one player's action triggers others to immediately follow. Meanwhile, Murto and Välimäki (2011) document dynamics that are similar to the equilibrium of this paper: players exit at a time-varying rate that rises when an opponent exits. That is, herding is not immediate but rather gradual and probabilistic. In both these settings, gradual herding occurs because information cascades would make delay strictly profitable. In Murto and Välimäki (2011), this is due to learning externalities: observing how opponents react to an exit allows for a more informed exit decision. In my setting it is because deterministic reports carry no credibility and thus are not profitable. However, my key departure is in introducing payoff externalities. I show that the strength of herding depends on these externalities, and that large preemptive motives can even induce anti-herding.

This is not the first paper to incorporate observational learning into a preemption game. In Hopenhayn and Squintani (2011) and Bobtcheff, Bolte, and Mariotti (2017), players learn about their opponents' propensity to act by observing how long they delay. Meanwhile, I consider a setting where players learn about a variable of common value, introducing learning externalities. Such has been studied by Moscarini and Squintani (2010), Bobtcheff, Lévy, and Mariotti (2025), and Chen, Ishida, and Mukherjee (2023). Moscarini and Squintani (2010) studies a winner-takes-all race where agents choose when to quit the race, documenting herding in the timing of actions, taking the form of simultaneous exits rather than one exit triggering others. In Bobtcheff et al. (2025), agents invest under bad-news learning, leading to a winner's curse and thus investment delays. Chen et al. (2023) also consider bad-news learning, finding that one firm's entry can induce another to immediately follow.

More generally, this paper contributes to the literature on games of preemption. In the classic application to technology adoption (Reinganum (1981), Fudenberg and Tirole (1985)) firms face a tradeoff: delaying adoption lowers costs but entails the risk of being preempted. In these papers, as is standard throughout the literature, the marginal benefit of waiting and the cost of preemption are exogenous. I depart from this by considering a setting where firms' payoffs depend on consumer beliefs, making them equilibrium objects. I show that the strategic tradeoff between the benefit of waiting and the cost of preemption can emerge endogenously, even when there is no exogenous benefit of delay. Moreover, firms can be rewarded for succeeding their rivals in a way that nullifies preemptive motives. That is, the cost of being preempted can be entirely offset in equilibrium.

Finally, the notion of faking arises in other work. In Boleslavsky and Taylor (2024), a single agent chooses whether to fake a project or wait for a valid one, and a principal decides whether to approve it. They similarly find that the agent generates fakes via a non-homogenous Poisson process, but that faking increases in the short run. The Poisson arrival of inaccurate information is also a feature of Che and Hörner (2018) and entails spamming by recommender systems. Meanwhile, in Gratton, Holden, and Kolotilin (2018) a sender chooses when to send a potentially fake report, where earlier reports are subject to greater scrutiny. Due to the role of scrutiny, they find that earlier reports are more credible in equilibrium.

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3, I characterize the equilibrium. In Section 4, I present the core economic implications of this equilibrium, which pertain to competition and herding. I present a welfare analysis and comparative statics in Section 5 and 6, respectively. Section 7 concludes. All proofs are relegated to the Appendix.

2. A model of breaking news

There are $N \geq 1$ firms, indexed by i, and one consumer. Time, which is continuous and has an infinite horizon, is denoted by $t \in [0, \infty)$. There exists some story, and the time-invariant state $\theta \in \{0, 1\}$ denotes whether it is true ($\theta = 1$) or false ($\theta = 0$). At $t = 0$, all players are endowed with a common prior $p_0 \equiv Pr(\theta = 1) \in (0, 1)$.

Learning and reporting Firms learn about θ via one-sided Poisson signals: if $\theta = 1$, a private signal revealing that $\theta = 1$ arrives to each firm at a Poisson rate $\lambda > 0$, where the time of this arrival is independent across firms.[4] Formally, letting $s_i \in [0, \infty]$ denote the time at which such a conclusive signal arrives to firm i, with $s_i = \infty$ denoting that a signal never arrives, $s_i \sim (1 - e^{-\lambda s_i})$ if $\theta = 1$, and $s_i = \infty$ if $\theta = 0$.

Firms choose whether and when to report the story. At any time t, a firm can make a report as long as they have not already done so. As the payoff function will illustrate, the content of this report can be interpreted as an assertion that the story is true, i.e., that $\theta = 1$. A report history H is a partially ordered set of pairs (i, t_i), pairing each firm i who has reported with a report time t_i, with elements ordered according to the order in which the reports were made.[5] All players observe the current report history.

Payoffs A firm who never reports earns a payoff of 0. A firm who does report earns

$$k_n \alpha - \beta \mathbb{I}[\theta = 0]. \tag{1}$$

The first term ($k_n \alpha$) is the market share (i.e., viewership or readership) that the firm enjoys from reporting a story. It is the product of k_n, a parameter capturing the role of the firm's order n, and α, the credibility of the report. More precisely,

[4] An extension where firms have heterogeneous learning abilities is included in the Supplemental Appendix.

[5] Formally, elements are ordered according to relation $\succsim$, where $(i, t_i) \succ (j, t_j)$ if $t_i > t_j$ or $t_i = t_j$ but i reported first, and $(i, t_i) \sim (j, t_j)$ if the reports were made simultaneously.

the index n denotes that the firm was the n^{th} to report: $n \equiv |H| + 1$, where H is the current history at the time of the report. I assume that $k_1 \geq k_2 \geq ... \geq k_N \geq 0$, i.e., firms who report earlier than their competitors earn greater market share, all else equal. Meanwhile, credibility α denotes the consumer's belief, at the time that the report is made, that the firm has independently confirmed the state. Formally, it is the belief that $s_i \in [0, t]$, where t is the time of the report. The second term of (1), $-\beta\mathbb{I}[\theta = 0]$, is the penalty of error: a firm who reports when $\theta = 0$ incurs a penalty $\beta > 0$. [6]

2.1. Discussion of Modeling Assumptions

Before proceeding, I briefly discuss the above modeling assumptions. There are three important features of the payoff environment: (1) firms are rewarded for preempting their opponents (2) payoffs depend on consumer beliefs, and (3) errors are costly. Regarding consumer beliefs, I specifically assume that market share depends on consumers' belief that the story has been independently verified and thus take the stance that consumers value news firms that do their due diligence. One may ask how crucial this specific definition of credibility is to the results. For instance, one could have instead defined credibility to be consumers' belief that the story is true. Under such an alternative specification, the key qualitative features of the equilibrium, with regards to herding and the copycat effect, would remain unchanged. However, the fact that market share is a function of consumer beliefs is crucial: if market share were to only depend on the order of the firm as in a standard preemption game, herding would never occur. [7] Meanwhile, the penalty of error captures the reputational harm from making a report that is later uncovered to be false, and disciplines firms for reporting unverified stories. The assumption that this penalty is exogenous is convenient, but not vital: for instance, if the penalty were decreasing in the total number of firms to err, herding would still occur.

Regarding the learning environment, there are two important assumptions: firms (1) observe private signals in favor of the story being true and (2) learn observationally. The conclusive, one-sided Poisson learning process can be interpreted as a setting where firms pursue reliable sources that can confirm

[6] For simplicity, I rule out the knife-edge case where for some n, $k_n = k_{n+1} \in (0, \beta)$.

[7] Both alternative specifications (where credibility is defined as the accuracy of the story and where market share is exogenous) are formalized in the Appendix.

a story. In the context of a terrorist attack, this could entail reaching out to contacts at the police department. The results do not hinge on precisely such a learning process: allowing firms to also observe signals confirming that the story is false with some small probability or signals that are highly informative but not conclusive would not qualitatively alter the equilibrium. However, it is vital that firms sometimes receive signals in favor of the story being true. If they were to only learn via Poisson signals confirming that the story is false (i.e., bad-news learning, as in Bobtcheff et al. (2025) and Chen et al. (2023)) a report could never indicate that the firm has received a signal which is consistent with the story being true, and thus equilibrium behavior would be fundamentally different. Finally, the assumption that firms receive independent signals isolates the role of observational learning and competition, rather than correlated signals, in giving rise to herding.

2.2. Equilibrium

A Markov[8] strategy F is a set of distributions $F_{p,n}$ over future report times for each belief $p \equiv Pr(\theta = 1)$ and order n of the next firm to report. [9] Specifically, the span of time the firm waits before reporting, conditional on not receiving a conclusive signal, is distributed according to a distribution $F_{p,n} \in \Delta[0, \infty]$ where ∞ denotes a lack of report. I restrict attention to symmetric equilibria and thus omit the firm's index in much of the analysis.

I place some restrictions on F. First, for analytical convenience, I assume that for all (p, n), $F_{p,n}$ is piecewise twice differentiable and right-differentiable everywhere on $[0, \infty)$. Second, I impose a selection criterion (SC): a firm immediately reports a story it knows is true. This is stated as Assumption 1.

Assumption 1 (SC). *F satisfies (SC) if $F_{1,n}(0) = 1$ for all $n \in \{1, ..., N\}$.*

This criterion rules out equilibria with periods of silence supported by off-path beliefs that reports made during these gaps have little credibility. In making this assumption, I am selecting an equilibrium rather than imposing restrictions on firms' behavior, namely, it is optimal for firms to abide by (SC) in equilibrium.

[8] In general, the Markov state should also include the identities of the remaining firms and beliefs over their private signals. However, it is without loss to define the state in this way within the class of strategy profiles that satisfy the below criteria, namely symmetry and (SC).

[9] If multiple firms report at the same history H, one firm will be assigned order n, another $n + 1$, etc., with their identities randomly determined according to a uniform distribution.

Conveniently, (SC) implies that fixing an n and starting belief p, all remaining players hold the same common belief about the state after t time has passed, assuming no new reports are made. This common belief is denoted by $p(t)$, and it follows from Bayes Rule that:

$$p(t) = \frac{pe^{-\lambda(N-n+1)t}}{pe^{-\lambda(N-n+1)t} + (1-p)}. \tag{2}$$

Defining strategies in this way, i.e. with a separate distribution for each (p, n), is convenient but introduces redundancy. Specifically, for any (p, n) and $t > 0$, $F_{p,n}$ and $F_{p(t),n}$ "overlap": both distributions specify the firm's reporting behavior at $(p(t+s), n)$ for any $s \geq 0$. Thus, I impose that the $F_{p,n}$ must be mutually consistent[10]: at any (p, n) and $t > 0$,

$$F_{p(t),n}(s) = \frac{F_{p,n}(t+s) - F_{p,n}(t_-)}{1 - F_{p,n}(t_-)} \text{ for all } s \geq 0 \text{ whenever } F_{p,n}(t) < 1, \tag{3}$$

where $F_{p,n}(t_-) \equiv \lim_{\tau \uparrow t} F_{p,n}(\tau)$. Let $\mathcal{F}$ denote set of distributions F that satisfy the above restrictions.

I define two terms to describe reporting: faking and truth telling. A report at time t is *fake* if it is made despite the firm lacking independent confirmation, i.e., $s^i \notin [0, t]$. Meanwhile, a report that is made after the firm has confirmation is *truthful*. Under the selection assumption (SC), strategies differ only in their distributions over fake reports.

I seek a symmetric Markov perfect equilibrium of this game. This is a Markov strategy F paired with beliefs α and p at each history such that F is sequentially rational and the beliefs are consistent with Bayes Rule. The consistency of α with Bayes Rule implies the following at all (p, n) on-path:

$$\alpha_n(p) = \begin{cases} \frac{\lambda p}{\lambda p + b_n(p)} & \text{if } F_{p,n}(0) = 0 \\ 0 & \text{if } F_{p,n}(0) > 0, \end{cases} \tag{4}$$

where $b_n(p) \equiv F'_{p,n}(0+)$, the right-derivative of $F_{p,n}$ at 0, is the instantaneous arrival rate of fake reports. This formula is intuitive. If $F_{p,n}(0) > 0$, there is a

[10] This is analogous to the *consistency condition* from Laraki, Solan, and Vieille (2005), who define this condition for a general class of continuous-time games of timing.

point mass of fake reports at (p, n), and because conclusive signals are distributed continuously over time, the instantaneous probability of a truthful report is zero. So, the consumer and all competing firms are certain that a report made at (p, n) was fake and thus assign to it zero credibility. If there is not a point mass of fake reports at (p, n), credibility is assessed by comparing the arrival rates of truthful reports (λp) to that of fake reports ($b_n(p)$), assigning higher credibility to reports made when the arrival rate of fake reports is relatively low.

3. Equilibrium characterization

This section presents the equilibrium characterization. I begin by defining the firm's problem and establishing two properties that are instrumental to the analysis. Then, as a steppingstone to the full model characterization, I consider the monopoly case. Finally, I characterize the equilibrium of the full model under competition.

3.1. The firm's problem

I now present the firm's problem. I begin by defining a useful object, the *first report distribution*. Fix a report history H and strategy profile F, and let p denote the common belief and n the order of the next firm to report. Index the firms who have not yet reported by i. The first report distribution $\Psi^i(s)$ denotes the probability that player i reported when or before s time has passed and was not preempted by any of the remaining firms (i.e., i was the first to make a new report). This is given by:

$$\Psi^i(s) = p \int_0^s e^{-\lambda t(N-n)} \prod_{j \neq i} (1 - F^j_{p,n}(t)) d(e^{-\lambda t}(F^i_{p,n}(t) - 1)) + (1-p) \int_0^s \prod_{j \neq i} (1 - F^j_{p,n}(t)) dF^i_{p,n}(t).$$

The firm's value from playing strategy F^i at (p, n) given each of its opponents plays F^j can then be written recursively as

$$V_{p,n}(F^i) = \int_0^\infty [k_n \alpha_n(p(s)) - \beta(1 - p^i(s))] d\Psi^i(s) + \sum_{j \neq i} \int_0^\infty V_{p^j(s), n+1}(F^i) d\Psi^j(s), \quad (5)$$

where $V_{p,N+1} \equiv 0$ and $p^j(s)$ denotes the common belief when s time has passed, conditional on no new reports having been made, except for a report by j at time s. The first integral of (5) is firm i's expected payoff from reporting conditional on being the first of the remaining firms to do so, and the second integral is conditional on being preempted. Specifically, upon being preempted by j at time s, the state changes from $(p(s), n)$ to $(p^j(s), n+1)$. Thus, the firm's continuation value upon being preempted is its value at this new state.

The firm's problem at (p, n) is

$$\max_{F^i \in \mathcal{F}} V_{p,n}(F^i).$$

3.2. Properties of equilibrium

I establish two necessary conditions on a firm's equilibrium strategy: (1) there cannot exist point masses in the distribution of fake reports and (2) whenever a firm is less-than-fully credible, it must satisfy certain indifference conditions. Similar properties arise in other games with continuous strategy spaces, where they result from competition.[11] Here, they are instead driven by the endogenous nature of credibility and thus hold even without competition.

First, let us establish that there are no point masses in faking.

Lemma 1. *In equilibrium, there are no point masses of fake reports: $F_{p,n}$ is continuous for any (p, n) such that $p < 1$.*

To see why this holds, recall that a report made when there is a point mass of faking yields zero credibility. Meanwhile, faking yields an expected penalty $\beta(1-p)$. Thus, a firm's value from faking at such a time is strictly negative and they could profitably deviate by truth telling: this would preclude them from making an error, ensuring a non-negative payoff.

Now let us establish the indifference property. To this end, let δ_s for $s \in [0, \infty]$ denote the distribution that places full mass on faking when s time has passed. Specifically, δ_0 denotes immediate faking, while δ_∞ denotes that the firm never fakes. Lemma 2 states that whenever credibility is less-than-perfect, the firm must find it optimal to: (1) fake immediately (play δ_0) (2) be truthful for some sufficiently short span of time dt and then fake (play δ_{dt}) and (3) never fake (play δ_∞).

Lemma 2. *In equilibrium if $\alpha_n(p) < 1$ and (p, n) is on-path, there exists an $\varepsilon > 0$ such that*

$$V_{p,n} = V_{p,n}(\delta_s) \text{ for all } s \in [0, \varepsilon) \cup \infty,$$

[11] These include war of attrition games (Hendricks, Weiss, and Wilson (1988)) and all-pay auctions (Baye, Kovenock, and De Vries (1996)).

where $V_{p,n}(\delta_s)$ is the value from playing δ_s at (p, n) and F at (q, m) for all q and $m > n$.

I will now provide some insight into the proof. Let us begin by considering why δ_{dt} must be optimal for $dt \in [0, \varepsilon]$. Given the regularity conditions on the firm's strategy, $\alpha_n(p(s))$ is right-continuous in s. So, if $\alpha_n(p) < 1$, $\alpha_n(p(s)) < 1$ for all s sufficiently small. Furthermore, whenever $\alpha_n(p(s)) \in (0, 1)$, the firm is faking with a strictly positive hazard rate. This means that the firm mixes between faking with delay in $[0, \varepsilon]$, implying that all such pure strategies are optimal. Next, let us consider why never faking must be optimal. If it were not, then a firm who has not received a conclusive signal must fake with probability 1. To achieve this, the firm must sustain a sufficiently high hazard rate of faking as t tends to ∞. But because the hazard rate of truthful reports tends to zero as p falls, credibility would tend to zero, making faking suboptimal.

3.3. The monopoly characterization

I now characterize the equilibrium under a monopoly, i.e., assuming $N = 1$. As there is only one firm, I drop the n index for the remainder of this subsection.

Claim 1. *Under a monopoly, for all p on-path: $\alpha(p) = \min\{\beta/k, 1\}$.*[12]

Claim 1 states that the monopolist's credibility is constant over time and not always perfect. That is, errors can occur when the ex-post penalty of error is relatively low.

To see why credibility must be constant, let us recall from Lemma 2 that a firm who fakes must be indifferent between faking immediately and after some wait dt. By the martingale property of firm's belief p, both of these strategies yield the same expected penalty from error $\beta(1 - p)$. For both strategies to be optimal, they must also yield the same expected market share $k\alpha$, and thus credibility must be constant. This reasoning is predicated on the fact that waiting is costless for a monopolist: not only is waiting intrinsically costless (there is no discounting), a monopolist also does not face the strategic cost to waiting that preemption entails. As I will later show, this strategic cost of waiting is precisely what gives rises to dynamics in credibility under competition.

Though a monopolist's credibility is constant, its strategy is dynamic: the hazard rate of faking (b) strictly decreases and tends to zero whenever credibility

[12] A proof is omitted, as this is a special case of the N-firm characterization that follows.

is less than one. This follows from (4) and the firm's learning process: the absence of a report means that the firm has not received a conclusive signal, causing the common belief to drift down. For credibility to remain constant, b must decline as well.

Now, let us consider why truth-telling cannot be sustained when $\beta/k < 1$, and why credibility is equal to β/k. Suppose by contradiction that $\beta/k < 1$ and the firm is truthful. This implies full credibility, and thus that the market share ($k\alpha(p)$) exceeds the penalty of error (β). So, it is strictly optimal to report, even if the story is false. That is, the firm can profitably deviate by faking. We conclude that in any equilibrium, the firm fakes with positive probability. To pin down the level of credibility, we recall from Lemma 2 that a firm who fakes must be indifferent between faking immediately and remaining truthful. Indeed, there is a unique value of credibility that ensures indifference: β/k. There is some intuition behind this: the bigger β/k is, the more costly errors are compared to market share for any α, and thus the more costly faking is relative to truth telling. So, α must correspondingly increase to maintain indifference.

3.4. Full model characterization

Now, I characterize the equilibrium of the full model (i.e., under N firms). I show that any equilibrium is the solution to a recursive set of boundary value problems.

Let us begin by deriving the conditions under which the firm is truthful. This both serves as a first step to a full characterization and illustrates how competition can exacerbate faking. Proposition 1 establishes two conditions which together are necessary and sufficient for truth telling. In addition to the condition that guaranteed truth-telling under a monopoly ($k_n \leq \beta$), competition requires the common belief to lie below some threshold.

Proposition 1. *In equilibrium, firms are truthful ($\alpha_n(p) = 1$) if and only if:*

1. $k_n \leq \beta$
2. $p \leq p_n^* \equiv \min\{\frac{k_n - \beta}{\frac{k_n}{N-n+1} - \beta}, 1\}$.

This second condition can be explained by the fact that under competition, faking and truth telling each pose a different kind of risk to the firm: while truth telling entails the risk of being preempted, faking entails the risk of making an error. Both depend on the belief p about the state: higher p implies a lower probability of error and a higher probability of preemption. The latter is due to the fact that preemption is more likely when the story is true, because an opponent reports not only when it is faking, but also when it has received confirmation. Since a lower risk of error and higher risk of preemption both make faking relatively more profitable, truth telling is harder to sustain when p is high.

It remains to characterize the firm's behavior when truth telling does not hold. To this end, I obtain a key result: when the firm fakes, credibility must satisfy an ODE and limit condition. As I illustrate below, these conditions are necessary to satisfy the firm's indifference conditions.

Proposition 2. *In equilibrium, at all (p, n) where firms are faking ($\alpha_n(p) < 1$), the following ODE is satisfied:*

$$\alpha_n'(p) = -\frac{1}{k_n(1-p)\alpha_n(p)}\frac{N-n}{N-n+1}[k_n\alpha_n(p) - V_{\tilde{p},n+1} - \beta(1-\alpha_n(p))(1-p)], \quad \text{(ODE)}$$

where $\tilde{p} \equiv \alpha_n(p) + (1-\alpha_n(p))p$.

Furthermore, $\lim_{p\to 0+}\alpha_n(p) = \beta/k_n$ if $k_n > \beta$ and $\lim_{p\to p_n^*+}\alpha_n(p) = 1$ if $k_n \leq \beta$.

Let us begin by considering why (ODE) must hold on the region where the firm is faking. Lemma 2 established that at any such time, there exists an $\varepsilon > 0$ such that the strategies δ_Δ yield the same payoff for all $\Delta \in (0, \varepsilon]$. This implies

$$\frac{d}{d\Delta+}V_{p,n}(\delta_\Delta)\bigg|_{\Delta=0} = 0. \tag{6}$$

It follows from (5) that

$$V_{p,n}(\delta_\Delta) = \int_0^\Delta k_n\alpha_n(p(s))d\Psi^i(s) + (N-n)\int_0^\Delta V_{p^{-i}(s),n+1}d\Psi^{-i}(s) + \\ (1 - \sum_j \lim_{s\to\Delta-}\Psi^j(s))[k_n\alpha_n(p(\Delta)) - \beta(1-p(\Delta))],$$

where Ψ is the first-report distribution associated with the strategy profile in which i plays δ_∞ and all $j \neq i$ play the equilibrium strategy $F_{p,n}$. Differentiating, we obtain

$$\lim_{\Delta\to 0+} \frac{d}{d\Delta} V_{p,n}(\delta_\Delta) = [\frac{dp}{dt}(k_n \alpha_n'(p)) - \frac{\lambda p(N-n)}{\alpha_n(p)}(V_{\tilde{p},n} - V_{\tilde{p},n+1})]. \quad (7)$$

Setting the right-hand side to zero, in accordance with (6), yields (ODE). Equation (7) illustrates that waiting to fake, rather than faking immediately, has two consequences for the firm's payoff. The first is that α_n, and thus the market share from reporting, may change. This rate of change is $\frac{dp}{dt}(k_n \alpha_n'(p))$. The second consequence is that the firm risks being preempted: this happens at a Poisson rate $\frac{\lambda p(N-n)}{\alpha_n(p)}$, in which case its expected payoff changes by $V_{\tilde{p},n} - V_{\tilde{p},n+1}$. I call this decrease in value the *regret* from preemption.

Let us examine the rate and regret of preemption more closely. As one might expect, the rate of preemption is increasing in the number of rival firms remaining ($N - n$) and the expected rate at which these rivals can confirm the story (λp). It is also decreasing in credibility: less credible firms are more likely to fake, and thus more likely to preempt. Meanwhile, the regret of preemption is the difference between two values, $V_{\tilde{p},n+1}$ and $V_{\tilde{p},n}$. $V_{\tilde{p},n+1}$ denotes the firm's continuation value in the event that it is preempted at (p, n). This value is taken at $(\tilde{p}, n+1)$ because preemption affects both the firms order and the common belief: while the common belief was p prior to the rival firm's report, it increases to $\tilde{p} \equiv \alpha_n(p) + (1 - \alpha_n(p))p$ in its immediate aftermath. This expression for $\tilde{p}$ demonstrates that a rival firm's report means one of two things: either the report was triggered by a conclusive signal, in which case the new belief should be 1, or it was fake, in which case the new report offers no new information and the belief remains p. Since faking is unobservable, the new common belief $\tilde{p}$ is an average of these two conditional beliefs, where the weight given to the report being informed is its credibility. Meanwhile, $V_{\tilde{p},n}$ denotes the continuation value conditional on not being preempted. This value is not assessed at the belief prior to preemption p, but rather the posterior $\tilde{p}$. In this sense, $V_{\tilde{p},n+1} - V_{\tilde{p},n}$ denotes firm's regret from not having reported after being preempted.

In addition to (ODE), Proposition 2 establishes that one of two limit conditions must hold. Which condition holds depends on the model parameters, and like (ODE), these conditions result from the firm's indifference condition. First consider the case where $k_n \leq \beta$. It follows from Proposition 1 that $\alpha_n(p) = 1$ whenever $p \leq p_n^*$. Thus, $\alpha_n(p)$ limits to 1 as the belief approaches p_n^*. Otherwise, there would be an upward discontinuity at p_n^*, meaning that at beliefs close to p_n^*, the firm could profitably deviate by waiting until p_n^* to fake, causing a failure of indifference. When $k_n > \beta$, the firm never truth tells, so by Lemma 2 it must always be indifferent between faking and truth telling. As p approaches zero, a firm who fakes does so being nearly certain that it will incur penalty β. Thus, the payoff from faking limits to the following:

$$\lim_{p \to 0+} V_{p,n}(\delta_0) = k_n \lim_{p \to 0+} \alpha_n(p) - \beta.$$

Meanwhile, the value from truth-telling limits to zero, as it becomes certain that the firm never receives a conclusive signal and thus never reports. So, the limit condition in this case, $\lim_{p \to 0+} \alpha_n(p) = \beta / k_n$, ensures that indifference holds in the limit.

To take stock, Proposition 1 and Proposition 2 provide two necessary conditions on equilibrium credibility. They pin down the region in which truth telling occurs (Proposition 1), and show that otherwise, credibility must satisfy a recursive boundary value problem (Proposition 2). One can show that these two conditions are sufficient for an equilibrium as well, provided that the firm's strategy is consistent with this credibility function. Thus, the equilibrium is fully characterized by the solution to a recursive set of boundary value problems. While I do not derive a closed-form solution to this problem, I use the Picard-Lindelöf theorem to establish existence and uniqueness. This result is stated as Theorem 1.

Theorem 1. *There is a unique equilibrium, where uniqueness applies on-path.*

4. Competition and Herding

I now present the main economic implications of the paper. First I show that competition gives rise to dynamics in credibility and exacerbates faking whenever firms face a preemptive motive. I then show that observational learning can induce herding via a *copycat effect*, wherein one report triggers a surge in faking. I establish that the nature of competition affects herding, as the copycat effect is more pronounced when the immediate gains to preemption are lower.

4.1. The effects of competition

I begin by discussing the equilibrium effects of competition. I consider two cases in turn: the first where the last firm is truthful ($k_N \leq \beta$) and the second where the last firm fakes with positive probability ($k_N > \beta$). While firms face a preemptive motive in the former case, this motive disappears in the latter case.

Proposition 3 shows that credibility strictly improves over time when $\beta > k_N$, as long as no new reports are made.

Proposition 3. *If $\beta > k_N$, credibility strictly increases ($\frac{d}{dt}\alpha_n(p(t)) > 0$) and faking strictly decreases ($\frac{d}{dt}b_n(p(t)) < 0$) over time whenever firms are not truthful.*

To understand this result, it is helpful to rewrite (ODE) in the following form:

$$\frac{d}{dt}\alpha_n(p(t)) = \frac{\lambda p(t)(N-n)}{\alpha_n(p(t))k_n}[V_{\tilde{p},n} - V_{\tilde{p},n+1}]. \tag{8}$$

We can see from (8) that credibility strictly increases if and only if there is a positive regret from preemption. There is intuition behind this: under a positive regret from preemption, if credibility remained constant, a firm would strictly benefit from faking immediately—this would allow the firm to avoid being preempted while leaving its credibility unchanged. To restore indifference, the firm must be compensated for waiting, which can only be achieved via increasing credibility. That is, credibility increases to mitigate the haste-inducing effects of preemption.

This being said, it is not necessarily true that there is a positive regret from preemption even when there are multiple firms in the market. However, there is a positive regret when $\beta > k_N$, i.e. when β is high enough to ensure the last firm is truthful. The reasoning for this is most easily illustrated in a duopoly setting ($N = 2$) where $\beta \in (k_2, k_1)$. In this case, a firm fakes with a positive hazard rate as long as nobody has reported yet, but switches to truth telling as soon as its opponent makes a report. To see why the credibility of the first report, α_1, must strictly increase over time, suppose instead that α_1 were constant as in the monopoly case. Since $k_1\alpha_1(p(t))$ must limit to β (Proposition 3), it follows that $k_1\alpha_1(p(t)) = \beta$ for all t. This implies a failure of indifference: the market share from reporting first is so high that faking is strictly optimal. So, the market share of the first firm must instead approach β from below. This restores indifference because it increases the value of truth-telling in two ways: (1) the lower market share from reporting first lowers the regret of being preempted and (2) increasing α provides an incentive to wait.

Proposition 3 also states that $b_n(p(t))$ is decreasing in t, an immediate corollary of the increasing nature of credibility. While this same result obtained under a monopoly, the strictly increasing nature of credibility implies that $b_n(p(t))$ decays more quickly than under the monopoly equilibrium. I.e., the firm's preemptive motive gives rise to more extreme dynamics in faking.

So far, we have restricted attention to the case where $k_N < \beta$. One can show that if this does not hold, preemption becomes costless in equilibrium and the dynamics in α_n disappear. Namely, credibility will be constant at a level where market share is not affected by the firm's order. That is, firms enjoy higher credibility from reporting after their opponents, which mitigates the decline in k_n in such a way that makes preemption costless. This is stated as Proposition 4.

Proposition 4. *If $k_N \geq \beta$, credibility is such that market share is unaffected by the time of the report or the firm's order. Formally, $\alpha_n(p) = \beta/k_n$ for all (p, n).*

To illustrate why this holds, it is again helpful to consider the duopoly case, now assuming that $\beta < k_2 < k_1$. In this case, the market share for the second reporter equals β and the market share of the first reporter must again limit to β but cannot limit to β from below. If it did, a firm could profitably deviate by being truthful: being preempted would *benefit* the firm, as it would yield a higher market share β. Instead, the market share of the first firm, $k_1\alpha_1(p)$, must equal β, which is exactly the market share for the second firm.

These results do not just pertain to equilibrium dynamics, they also speak to the effects of competition on faking. To formalize this, let $\bar{b}_n$ denote equilibrium faking under a monopoly ($N = 1$) with maximal market share k_n. This denotes equilibrium faking in the absence of competition. Corollary 1 states that there is more faking under competition if and only if $\beta > k_N$.

Corollary 1. *Faking is strictly higher under competition when $\beta > k_N$ (provided the firm is not truthful) and unaffected by competition when $\beta \leq k_N$. I.e., for all (p, n), $b_n(p) \geq \bar{b}_n(p)$, where the inequality holds strictly if and only if $b_n(p) > 0$ and $\beta > k_N$.*

This follows directly from the previous two propositions. Proposition 3 established that credibility limits to the monopoly value from below, whereas Proposition 4 showed that credibility is equal to the monopoly value when $\beta \leq k_N$. Thus, faking is exacerbated by competition, but only when β is sufficiently high. When the cost of error is sufficiently low, competition has no effect on faking.

Let us take stock of these results. Proposition 3 asserts that news reports that are made with greater delay can be perceived as more trustworthy to consumers.

I.e., consumers have greater trust in a firm's journalistic standards when a report is not made quickly. In this sense, this model provides a justification for consumer distrust of hasty reporting that originates from the firm's preemptive motive. Meanwhile, Proposition 4 shows competition alone does not imply preemptive motives: payoffs sometimes endogenously adjust in such a way that makes preemption costless. Together these results imply that competition exacerbates faking, but only when the consequences of making an error are sufficiently high.

4.2. Herding

The previous subsection described the dynamics of reporting conditional on no new reports being made. However, credibility and faking will also change in response to a new report. To illustrate this, it is helpful to plot simulations over the course of time. Figure 1 does this for the case when $k_N < \beta$. Credibility increases and faking decreases as long as no new reports are made (as per Proposition 3), but new reports trigger discrete changes in both variables. Notably, these discrete changes are non-monotonic: while the first four reports trigger an increase in faking (and decrease in credibility), the fifth report leads to a decrease in faking (and increase in credibility). Meanwhile, Figure 2 simulates the case where $k_N \geq \beta$. In this case, credibility is flat conditional on no new reports (as per Proposition 4), with new reports triggering exclusively upwards jumps in credibility and faking.

$$C_n(p) \equiv b_{n+1}(\tilde{p}) - b_n(p).$$

Both these simulations illustrate the *copycat effect* in which a new report triggers an increase in faking. The first simulation also demonstrates *anti-herding*, where a new report triggers a decrease in faking. I define both phenomena formally below. To this end, let us first define the change in faking triggered by a report at (p, n) as $C_n(p)$:

Definition 1. For $n < N$, a report at (p, n) triggers the *copycat effect* if $C_n(p) > 0$, and triggers *anti-herding* if $C_n(p) < 0$.

To understand when and why these phenomena occur, it is helpful to decompose the change in faking into two components. To this end, let us recall that a new report affects two changes to the state. First, it increments the order of the next firm to report from n to $n + 1$. Second, firms learn observationally from the report,

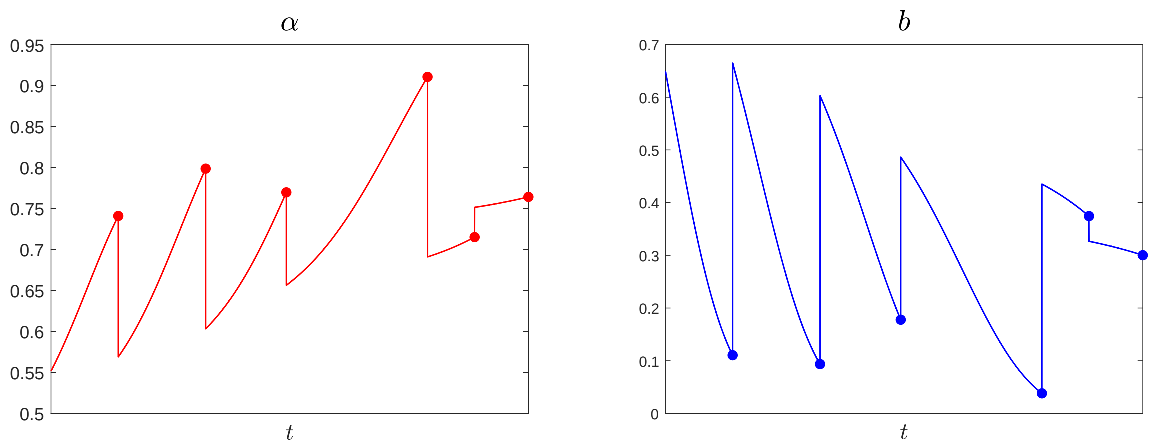

Figure 1: Simulation of credibility (α) and the hazard rate of faking (b), over the course of a game when $k_N < \beta$. Upwards jumps in b illustrate the copycat effect, while downwards jumps illustrate anti-herding.

and thus the common belief increases from p to $\tilde{p}$. The following decomposition isolates the respective impacts of these two changes on faking:

$$C_n(p) = \underbrace{[b_{n+1}(p) - b_n(p)]}_{\text{change in order}} + \underbrace{[b_{n+1}(\tilde{p}) - b_{n+1}(p)]}_{\text{change in belief}}.$$

In equilibrium, the change in belief will always cause faking to increase. I.e., observational learning always contributes positively to the copycat effect. This is stated as Corollary 2.

Corollary 2. *The change in belief from a new report increases faking: for any $n < N$, $b_{n+1}(\tilde{p}) - b_{n+1}(p) \geq 0$, where the inequality holds strictly when $b_{n+1}(\tilde{p}) > 0$.*

This is an immediate corollary of Proposition 3 and Proposition 4, which establish that whenever a firm fakes, $b_n(p(t))$ is decreasing in t, or equivalently that $b_n(p)$ is increasing in p. That is, an increase in the common belief implies an increase in faking. There is intuition for this as well: all else equal, a higher p implies a greater incentive to fake because this corresponds to a lower risk of error and higher risk of preemption, which both make faking relatively more valuable.

However, the change in order has an ambiguous effect on faking, i.e., $b_{n+1}(p) - b_n(p)$ may be positive or negative depending on the payoff parameters. To illustrate this, it is helpful to study two contrasting examples. First, consider a three-firm

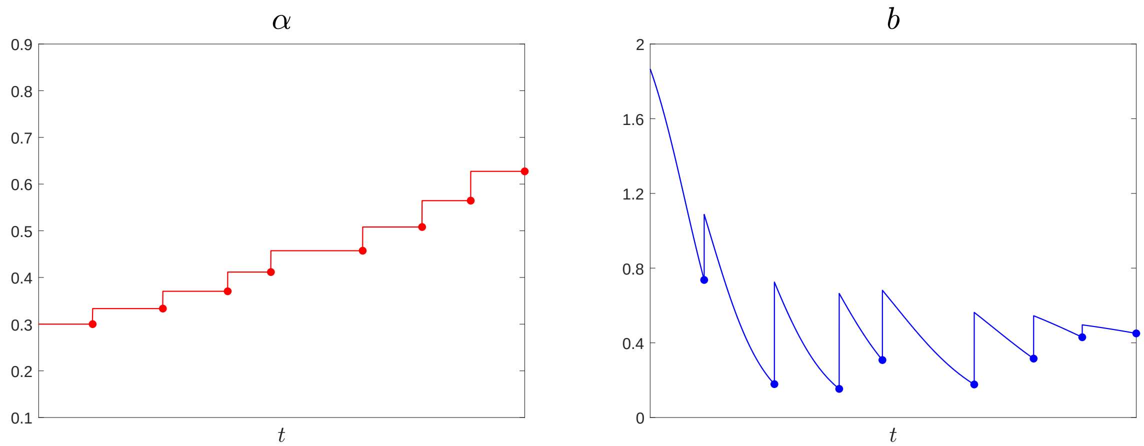

Figure 2: Simulation of credibility (α) and the hazard rate of faking (b), over the course of a game when $k_N > \beta$.

setting $(N = 3)$ where $k_1 > k_2 \approx k_3$ and $\beta \in (k_3, k_1)$. In this case, firms have a preemptive motive as long as nobody has yet reported, but this motive all but disappears once a report has been made since order will negligibly impact a firm's market share. So, a change in order reduces the incentive to fake: $b_2(p) - b_1(p) < 0$. Next, let us consider the same example but now assuming that $k_1 = k_2 > k_3$. All else equal, the first and second firm to report enjoy the same market share. So, firms face no cost to preemption as long as nobody has reported yet. Instead, this cost appears as soon as the first report is made. So in this case, a change in order increases the incentive to fake: $b_2(p) - b_1(p) > 0$.

These examples illustrate that the change in faking induced by a new report depends on the preemptive motive of the current reporter as compared to later reporters. Specifically, it indicates that the copycat effect may be more likely and more pronounced when firms face a *lower* immediate preemptive motive. I generalize this as Proposition 5, where I also show that both the copycat effect and anti-herding may occur in equilibrium depending on the relative size of this preemptive motive. This result states that the copycat effect is both more likely to occur and more pronounced when the immediate market share from reporting is lower. Furthermore, the copycat effect must occur when k_n is low, whereas when k_n sufficiently high, the negative effect of the change in order on $C_n(p)$ outweighs the positive effect of observational learning and leads to anti-herding.

Proposition 5. *For any p and $n < N$, $C_n(p)$ is strictly decreasing in k_n when $b_n(p) > 0$. Furthermore, fixing all other parameters and a p, there exists a $\overline{k}$ such that the copycat effect occurs if $k_n < \overline{k}$ and anti-herding occurs if $k_n > \overline{k}$.*

This result is driven by the fact that lowering k_n has two effects on equilibrium incentives. Directly, it lowers the preemptive motive of the current (n^{th}) reporter compared to later reporters. This yields the current firm less likely to fake, and thus more credible. This then has an indirect effect on faking for later reporters via observational learning. Namely, if the n^{th} firm is more credible, the common belief $\tilde{p}$ will be higher in the immediate aftermath of a report. This implies a higher probability of being preempted and a lower expected penalty of error, yielding the following firm more willing to fake. Thus, lowering k_n exacerbates the copycat effect by both reducing immediate faking and increasing faking in the aftermath of a new report. Furthermore, if k_n is sufficiently large, anti-herding will occur: as k_n rises, faking increases unboundedly for the n^{th} firm, and because the credibility of this report tends to zero, faking in its aftermath falls.

The above result illustrates the role of payoff externalities in determining whether and the extent to which the copycat effect occurs. The common belief at the time of report is relevant as well, and it is for this reason that the cutoff $\overline{k}$ on k_n, which determines whether the copycat effect or anti-herding occurs, is contingent on p. One may then ask whether there exist conditions on p such that the copycat effect occurs given arbitrary payoff parameters. Corollary 3 establishes that it will always occur when p is sufficiently low.

Corollary 3. *Suppose $n < N$ and $k_{n+1} > \beta$. There exists a $\overline{p} > 0$ such that for all $p < \overline{p}$, the copycat effect occurs.*

This result is driven by the fact that the magnitude of observational learning, $\tilde{p} - p$, is decreasing in the starting belief p. This is true for two reasons. First, a high pre-report belief p leaves little room for the belief to increase further. Second, reports made when p is high are less credible, and thus have less impact on the common belief. This negative correlation between the common belief and observational learning means that the positive effect of observational learning on faking is salient when p is low. Indeed, when p is sufficiently small, observational learning is substantial enough to counteract the ambiguous effect of changing order and give rise to the copycat effect.

Let us now interpret and discuss the testable implications of these results. The copycat effect has important implications for the behavior of firms: in the aftermath of an opponent report, a firm may be more likely to report not because they have also verified the story, but because they are faking. So, the copycat effect is consistent with errors propagating through the market, for which there is some anecdotal evidence (e.g., the reporting surrounding the Boston Bombings and 2000 US Presidential Election). Furthermore, because under the copycat effect firm faking increases immediately and then starts its gradual decline, a new report is most likely in the immediate aftermath of an opponent report. That is, firms also herd on the *timing* of reports, both correct and erroneous ones. Herding in the timing of news is empirically documented by Cagé et al. (2020), and the copycat effect demonstrates that the interaction between strategic motives and social learning can explain this behavior.

However, the copycat effect does not always occur, and Proposition 5 shows that the nature of competition plays an important role in determining whether it happens and how pronounced it is. Specifically, this result indicates that it should be more prevalent when the gains to being an early reporter are relatively low. Notably, this would imply that herding and the propagation of errors should be more pronounced in settings where the benefit from being first are modest (e.g., where consumers multi-home less across news firms). Meanwhile, while errors may be more likely to arise in the first place when preemptive motives are high (e.g., when consumers follow multiple news firms), the fact that anti-herding is more probable in such settings implies that existing errors should be less likely to propagate to other firms.

5. Welfare analysis

In this section, I consider the welfare implications of competition. I show that competition reduces consumer welfare when preemptive motives are high but also when firms face small preemptive motives. To this end, I begin by formally modeling consumers in a way that microfounds the firm's market share.

5.1. Modeling consumers

I model consumers who multi-home across news firms and have a preference for news that has been verified. Let $\mathcal{N} \equiv \{1, ..., N\}$ denote the set of news firms.

Suppose there is a mass $K > 0$ of consumers, indexed by x. Each x subscribes to some subset $S_x \subseteq \mathcal{N}$ of the firms, which is the consumer's *subscription set*. For any $S \subseteq \mathcal{N}$, let $m(S)$ denote the mass of consumers such that $S_x = S$, where $\sum_{S \in 2^{\mathcal{N}}} m(S) = K$. The mass of consumers with a given subscription set does not depend on the identity of the firms within that set, but only on the number of firms in the set. Formally, for all $j \in \{1, ..., N\}$, there exists $\gamma_j \geq 0$ such that $m(S) = \gamma_j$ whenever $|S| = j$. The γ_j describe how much consumers multi-home across news firms: γ_j that are relatively large for low (high) j capture scenarios where there is low (high) multi-homing.

A consumer reads a story if he both *considers* the story and finds it optimal to read it. Consumer x considers the story if and only if he (1) subscribes to the firm ($i \in S_x$) and (2) has not previously considered the story ($j \notin S_x$ for all j that reported before i). The mass of consumers who consider firm i's story given that they were the n^{th} firm to report is

$$\sum_{j=1}^{N-n} \binom{N-n}{j} \gamma_j \equiv k_n.$$

A consumer's payoff from not reading a story is 0, while the payoff from reading i's story is

$$2\mathbb{I}(s_i \leq t) - 1 + u_x,$$

where u_x denotes the agent's intrinsic utility from consuming news, t is the time of the report and s_i denotes the time at which the firm receives a Poisson signal confirming the story. The first component of this payoff ($2\mathbb{I}(s_i \leq t) - 1$) formalizes the notion that consumers value news that it has been independently verified by the firm. For any x, u_x is distributed uniformly on $[-1, 1]$ and is independent of the agent's subscription set. Thus, agents are heterogenous in their intrinsic preference for news, with some who find it beneficial while others find it costly. Consumers maximize expected utility, and thus x will read the story if and only if $\alpha \geq c_x$. It follows that i's market share is $k_n \alpha$. We define (consumer) welfare to be the expected utility of a consumer.

5.2. Competition and welfare

Now, I compare welfare under competition to that under a competition-free benchmark. Given parameters $(\lambda, \beta, \{k_n\}_{n=1}^{N})$ with N firms, the *competition-free* *benchmark* is a setting with one firm and parameters $(N\lambda, \frac{K}{k_1}\beta, K)$, where $K \equiv \sum_{n=1}^{N} k_n$. This benchmark considers a single firm, holding fixed the market's total ability to learn and total market share. The penalty of error is scaled to ensure that its ratio to market share is the same as that of the first firm under competition.[13]

Proposition 6 establishes that welfare is higher under competition both when the benefit to being the first reporter is sufficiently high and when the firm's order is sufficiently unimportant for its payoff. For analytical convenience, I assume away the knife-edge case where $k_1 = \beta$.

Proposition 6. *Suppose $N \geq 2$ and $k_1 > \beta$. Welfare is lower under competition if either:*

- *preemptive motives are large: $k_2 \in (0, \overline{k})$ for some $\overline{k} > 0$, or*
- *preemptive motives are small: $k_n > \underline{k}$ for all $n > 1$ and some $\underline{k} \in (0, k_1)$.*

In the previous section, we showed that when gains to being the first reporter are high, competition has two opposing effects on news reporting: it increases the probability that the first reporter fakes, but reduces the probability succeeding firms do so. Proposition 6 establishes competition nonetheless hurts consumer welfare in such settings. This is because a large benefit to being the first reporter (i.e., k_1 is large compared to k_2), corresponds to high levels of multi-homing (γ_N is high), meaning that a large proportion of consumers only consider the first report. Since the first report is less credible under competition, consumer welfare suffers: even though subsequent reports may be more credible, most consumers observe and consume the first report. When the order in which firms report is not significant for their payoffs, competition still harms welfare but for a different reason: the copycat effect. In this case, while competition has a negligible impact on the probability that fake reports are originated, the copycat effect leads succeeding firms to fake with a higher probability. This higher incidence of faking for succeeding firms reduces welfare among consumers who observe their reports. Thus, even when payoff externalities are insignificant, competition compromises consumer welfare via the copycat effect.

[13] One could have alternatively scaled the penalty of error by the number of firms N, and the below welfare comparison would still hold.

6. Comparative statics

I now consider how the equilibrium changes with the cost of error (β), the rate of learning (λ) and the number of firms (N). I discuss each parameter in turn.

6.1. Cost of error (β)

Fixing a state (p, n), a firm is more credible and fakes less under a high β. This result is stated as Comparative Static 1.

Comparative Static 1. *In equilibrium, $\alpha_n(p)$ is increasing in β and $b_n(p)$ is decreasing in β for any (p, n), and strictly so whenever $\alpha_n(p) < 1$.*

A higher β makes faking more costly for the firm. Thus, increasing β will either induce the firm to resort to truth telling, or to restore indifference, require that it is compensated for this costlier faking with higher credibility. In either case, this implies higher credibility and lower faking. This result suggests that costlier errors can improve the quality of information news firms provide. Perhaps more surprisingly, a higher β can be beneficial for firms as well. Indeed, in a winner-takes-all setting, firms' ex-ante value in equilibrium is increasing in β. This is stated as Corollary 4.

Corollary 4. *In a winner-takes-all setting, a firm's ex-ante value, $V_{p_0,1}$, is increasing in β and strictly so whenever firms fake with positive probability.*

While firms are unable to commit to truthful reporting, a higher β serves a similar role in equilibrium, curbing faking and thus improving credibility. As Corollary 4 demonstrates, credibility improves so much that it outweighs the increased cost of error, leaving firms better off.

6.2. Speed of learning (λ)

A higher λ has no effect on credibility, and increases faking, under any state (p, n). However, increasing λ also speeds up the decay in the common belief, and thus credibility improves as a function of time.

Comparative Static 2. *In equilibrium, $\alpha_n(p)$ is constant in λ, while $b_n(p)$ is weakly increasing in λ for any (p, n). Meanwhile, $\alpha_n(p(t))$ is increasing in λ for any $t > 0$.*

λ does not affect $\alpha_n(p)$ because it does not enter the boundary value problem which dictates the firm's credibility. However, an increase in λ does cause an increase in the arrival rate of truthful reports. Thus, $b_n(p)$ increases to maintain a constant level of credibility. While λ does not affect $\alpha_n(p)$, it does affect the common belief $p(t)$: under a high λ, firms learn about the state more quickly and thus will hold a lower belief at any time $t > 0$ condition on not receiving a conclusive signal. This lower belief implies a higher expected cost of erring, which makes faking more costly which must be counterbalanced by higher credibility at every $t > 0$.

6.3. Number of firms (N)

Finally, I study the effect of a change in the number of firms. This exercise sheds light on how firm entry can affect the quality of news. Comparative Static 3 establishes that firm entry deteriorates the credibility of the first report, and increases faking, but only if the report is made sufficiently early. In fact, market entry can improve credibility for reports made with sufficient delay.

Comparative Static 3. *A higher N lowers credibility for early reports. I.e., there exists $\underline{t} > 0$ such that equilibrium $\alpha_1(p_0(t))$ is decreasing in N, and strictly so if $\alpha_1(p_0) < 1$, for all $t < \underline{t}$.*

This result can be understood by noting that an additional firm affects two separate changes to the market. First, each firm faces more competition, and thus greater preemption risk. Second, the market has a higher ability to learn observationally, and thus the common belief decays more quickly in the absence of a report. The effect of an additional firm can be understood as the combination of these two changes: higher competition, which deteriorates credibility, and a greater ability to learn, which per Comparative Static 2 improves credibility. An increase in learning ability has a negligible impact on credibility for early reports because firms learn gradually, and it thus takes time for differences in learning to impact beliefs. However, an increase in competition will have a non-negligible impact on credibility even when $t = 0$. Thus, the impact of higher competition dominates when t is small, resulting in a net reduction in credibility. As time passes and the effect of faster learning grows, a reversal may take place, i.e., credibility may improve.

7. Conclusion

This paper presents a dynamic model of breaking news that accounts for the strategic and learning environment of the news market. Firms face payoff and learning externalities, which can exacerbate reporting errors and introduce dynamics in reporting behavior in distinct ways. The preemptive motive firms encounter causes errors by incentivizing hasty reporting and is responsible for lower news credibility which gradually improves over time. Meanwhile, observational learning propagates existing errors through the market via a copycat effect, where a report by one firm induces an immediate and persistent surge in faking by other firms, behavior that is consistent with clustering in the timing of news reporting. Crucially, this herding is not only driven by observational learning, but also by its interaction with the preemptive motive that firms face, with the copycat effect being more prevalent in settings with lower benefits to being a first reporter.

More generally, this paper sheds light on how payoff and learning externalities can interact in a game of timing. The nature of this interaction in more general environments warrants further investigation. Beyond this, this paper illustrates how the core tradeoff in games of preemption—between the strategic benefit of preemption and the non-strategic benefit of delay—can arise as an equilibrium phenomenon even when otherwise absent. Understanding when such phenomena may occur in games of timing is another avenue of future work.

Appendix

Before proving Proposition 1, I state a useful, technical lemma (Lemma 3), the proof of which is presented in the Supplemental Appendix.

Lemma 3. *$\alpha_n(p(s))$ is continuous in s for all (p, n) on path such that $s > 0$.*

Proof of Proposition 1. I begin by showing that $\alpha_n(p) = 1$ whenever $k_n < \beta$ and $p \leq p_n^* \equiv \frac{k_n - \beta}{k_n/n - \beta}$. To this end, fix an n, and suppose that $k_n < \beta$. I first show that for all $q < \frac{\beta - k_n}{\beta}$, $\alpha_n(q) = 1$. For all such q

$$V_{q,n}(\delta_0) = k_n \alpha_n(q) - \beta(1-q) \leq k_n - \beta(1-q) < k_n - \beta(1 - \frac{\beta - k_n}{\beta}) = 0.$$

Since $V_{q,n} \geq V_{q,n}(\delta_\infty) \geq 0$, it follows that $V_{q,n} > V_{q,n}(\delta_0)$. Thus, by Lemma 2, $\alpha_n(q) = 1$. Now, let $q_n^* \equiv \sup\{p | \alpha_n(q) = 1 \text{ for all } q < p\}$. It follows from the above that $q_n^* \geq \frac{\beta - k_n}{\beta}$. I claim that $q_n^* \leq p_n^*$. Suppose by contradiction that $q_n^* < p_n^*$. By Lemma 3, there exists an $\varepsilon > 0$ such that for all $p \in (q_n^*, q_n^* + \varepsilon)$, $\alpha_n(p) < 1$, and thus $V_{p,n} = V_{p,n}(\delta_0) = k_n \alpha_n(p) - \beta(1-p)$. So, $\lim_{p \to q_n^*+} V_{p,n} = k_n - \beta(1 - q_n^*)$. By definition of V, because by Lemma 1 $F_{p,n}$ is absolutely continuous, it follows that $V_{p,n}(\delta_\infty)$ is as well, and thus: $\lim_{p \to q_n^*+} V_{p,n}(\delta_\infty) = V_{q_n^*,n}(\delta_\infty) = \frac{k_n q_n^*}{n}$. In order for δ_∞ to not serve as a profitable deviation for $p \in (q_n^*, q_n^* + \varepsilon)$, it must be that for all such p, $V_{p,n}(\delta_0) \geq V_{p,n}(\delta_\infty)$. Taking a limit we obtain that $\lim_{p \to q_n^*+} V_{p,n}(\delta_0) \geq \lim_{p \to q_n^*+} V_{p,n}(\delta_\infty)$. Substituting the limits for $V_{p,n}$ and $V_{p,n}(\delta_\infty)$ above, we obtain that $\frac{k_n q_n^*}{n} \leq k_n - \beta(1 - q_n^*)$. However, $k_n \leq \beta$ and $q_n^* < p$ implies that $\frac{k_n q^*}{n} > k_n - \beta(1 - q^*)$. Contradiction.

Next, we show that $\alpha_n(p) < 1$ whenever $\beta \leq k_n$ or $p > p_n^*$. To this end, assume $\beta \leq k_n$ or $p > p_n^*$. Assume by contradiction that $\alpha_n(p) = 1$. Also assume by induction that if $n < N$, then the statement holds for $n + 1$. First, consider the case where $\alpha_n(q) = 1$ for all $q < p$. By (4), this implies that $F'_{q,n}(0) = 0$ for all $q < p$. Furthermore, by Lemma 1, this implies that $F_{p,n}(s) = 0$ for all $s > 0$, i.e., $F_{p,n} = \delta_\infty$. However, $V_{p,n}(\delta_0) = k_n - \beta(1-p) > \frac{k_n p}{n} = V_{p,n}(\delta_\infty)$, where the strict inequality follows from the assumption that either $\beta \leq k_n$ or $p > p_n^*$. Contradiction.

Next, consider the case where $\alpha_n(q) < 1$ for some $q < p$. By Lemma 3, for all $\varepsilon > 0$ sufficiently small, there exists $\overline{p} < p$ and $\overline{s} > 0$ such that $\alpha_n(\overline{p}) \in (1 - \varepsilon, 1)$ and $\alpha_n(q)$ is strictly increasing on $[\overline{p}(\overline{s}), \overline{p}]$. By Lemma 2, there exists some $\Delta \in (0, s)$

such that

$$V_{\overline{p},n}(\delta_\Delta) = V_{\overline{p},n}(\delta_0). \tag{9}$$

By definition,

$$V_{\overline{p},n}(\delta_\Delta) = \int_0^\Delta k_n\alpha_n(\overline{p}(s))d\Psi^i(s) + (N-n)\int_0^\Delta V_{\overline{p}^i(s),n+1}d\Psi^{-i}(s) +$$
$$(1-\sum_j \Psi^j(\Delta))[k_n\alpha_n(\overline{p}(\Delta)) - \beta(1-\overline{p}(\Delta))],$$

where Ψ is the first-report distribution associated with the strategy profile where i plays δ_Δ and all $j \neq i$ play $F_{p,n}$. Meanwhile,

$$V_{\overline{p},n}(\delta_0) = \int_0^\Delta k_n\alpha_n(\overline{p})d\Psi^i(s) + (N-n)\int_0^\Delta k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}^i(s))d\Psi^{-i}(s)$$
$$+ (1-\sum_j \Psi^j(\Delta))(k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}(\Delta)).$$

Thus, in order for (9) to hold, for some $r \in (0, \overline{s})$,

$$k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}^i(r)) < V_{\overline{p}^i(r),n+1}. \tag{10}$$

First, consider the case where $\alpha_{n+1}(\overline{p}^i(r))) < 1$. Then, for $\varepsilon > 0$ sufficiently small

$$V_{\overline{p}^i(r),n+1} = V_{\overline{p}^i(r),n+1}(\delta_0) = k_{n+1}\alpha_{n+1}(\overline{p}^i(r)) - \beta(1-\overline{p}^i(r)) < k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}^i(r)),$$

Thus, equation (10) is violated. Contradiction. Next, consider the case where $\alpha_{n+1}(\overline{p}^i(r)) = 1$ and $\beta < k_n$. By the inductive assumption, it follows that $\alpha_{n+1}(q) = 1$ for all $q \leq \overline{p}^i(s)$. Thus, $F_{\overline{p}^i(s),n+1} = \delta_\infty$. So, we have that for ε sufficiently small:

$$V_{\overline{p}^i(r),n+1} = \frac{k_{n+1}\overline{p}^i(r)}{N-n} \leq \overline{p}^i(r)k_n\alpha_n(\overline{p}) + (1-\overline{p}^i(r))k_n\alpha_n(\overline{p}) - \beta) = k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}^i(r)).$$

Again, this is a contradiction of (10).

Finally, consider the case where $\alpha_{n+1}(\overline{p}^i(r)) = 1$ and $\beta \geq k_n$. Recall that $\alpha_n(q) = 1$ for all $q \geq p_n^*$. Thus, because $\alpha_n(\overline{p}) < 1$, it follows from (3) that $\alpha_n(\overline{p}(s))$ must be strictly increasing in s for some $s > r$. Let

$r' \equiv \inf\{s > r | \alpha_n(\overline{p}(s))$ is strictly increasing$\}$. First, I claim that

$$k_n \alpha_n(\overline{p}(r')) - \beta(1 - \overline{p}^i(r')) < V_{\overline{p}^i(r'),n+1}. \tag{11}$$

By the inductive assumption, since $\alpha_{n+1}(\overline{p}^i(r)) = 1$, it must be that $\alpha_{n+1}(q) = 1$ for all $q < \overline{p}^i(r)$. Because $\alpha_n(\overline{p}(s))$ is weakly decreasing in s for $s \in [r, r']$, it follows by definition of $\overline{p}^i(s)$ that $\overline{p}^i(s) < \overline{p}^i(r)$ for all $s \in [r, r']$. Thus, for all $s \in [r, r']$ $V_{\overline{p}^i(s),n+1} = \frac{k_{n+1}\overline{p}^i(s)}{N-n}$. Then, for all $s \geq r$,

$$k_n \alpha_n(\overline{p}(s)) - \beta(1 - \overline{p}^i(s)) < V_{\overline{p}^i(s),n+1} \Leftrightarrow \overline{p}^i(s) < \frac{\beta - k_n \alpha_n(\overline{p}(s))}{\beta - k_{n+1}/(N-n)}.$$

Now, because $\alpha_n(\overline{p}(s))$ is strictly decreasing on $s \in [0, r]$,

$$k_n \alpha_n(\overline{p}(r)) - \beta(1 - \overline{p}^i(r)) < k_n \alpha_n(\overline{p}) - \beta(1 - \overline{p}^i(r)) < V_{\overline{p}^i(r),n+1}.$$

where the second inequality holds for the same reason as (10). Thus we have

$$\overline{p}^i(r') < \overline{p}^i(r) < \frac{\beta - k_n \alpha_{n+1}(\overline{p}(r))}{\beta - k_{n+1}/(N-n)} < \frac{\beta - k_n \alpha_{n+1}(\overline{p}(r'))}{\beta - k_{n+1}/(N-n)},$$

which implies (11).

It follows from this that there exists an $r'' > r'$ such that for all $s \in [r', r'']$, $\alpha_n(\overline{p}(s))$ is weakly decreasing and $V_{\overline{p}^i(s),n+1} > k_n \alpha_n(\overline{p}(r')) - \beta(1 - p^i(s))$. I now claim that $V_{\overline{p}(r'),n}(\delta_0) < V_{\overline{p}(r'),n}(\delta_{r''-r'})$. To see why, note that by definition,

$$\begin{aligned} V_{\overline{p}(r'),n}(\delta_{r''-r'}) - V_{\overline{p}(r'),n}(\delta_0) = &\int_{r'}^{r''} k_n[\alpha_n(p(s)) - \alpha_n(p(r'))] d\Psi^i(s) + \\ &\int_{r'}^{r''} [V_{p^i(s),n+1} - (k_n \alpha_n(p(r')) - \beta(1 - p^i(s)))] d\Psi^{-i}(s) \\ &+ \sum_j (\Psi^j(r'') - \Psi^j(r')) k_n(\alpha_n(p(r'')) - k_n \alpha_n(p(r'))). \end{aligned}$$

Since $\alpha_n(p(s)) \geq \alpha_n(p(r'))$ and $V_{p^i(s),n+1} > k_n \alpha_n(p(r')) - \beta(1 - p^i(s))$ for all $s \in [r', r'']$, it follows that $V_{\overline{p}(r'),n}(\delta_{r''-r'}) - V_{\overline{p}(r'),n}(\delta_0) > 0$. This contradicts Lemma 2. $\square$

Proof of Proposition 2. Proof by induction. Fix an n, and assume that $\alpha_m(p)$ satisfies the above for all $m > n$ such that (p, m) is on-path. I begin by showing that (ODE) must hold whenever $\alpha_n(p) < 1$. To this end, assume that $\alpha_n(p) < 1$. By Lemma 2, there exists an $\varepsilon > 0$ such that for all $\Delta \in (0, \varepsilon)$,

$$\frac{V_{p,n}(\delta_\Delta) - V_{p,n}(\delta_0)}{\Delta} = 0. \tag{12}$$

By definition, $V_{p,n}(\delta_0) = k_n\alpha_n(p) - \beta(1-p)$. Meanwhile,

$$V_{p,n}(\delta_\Delta) = \int_0^\Delta k_n\alpha_n(p(s))d\Psi^i(s) + (N-n)\int_0^\Delta V_{p^{-i}(s),n+1}d\Psi^{-i}(s) + \\ (1 - \sum_j \lim_{s\to\Delta-} \Psi^j(s))[k_n\alpha_n(p(\Delta)) - \beta(1-p(\Delta))],$$

where Ψ is the first-report distribution associated with the strategy profile in which i plays δ_∞ and all $j \neq i$ play the equilibrium strategy $F_{p,n}$. Specifically, for all $s > 0$,

$$\Psi^i(s) = p\lambda \int_0^s e^{-\lambda r(N-n+1)}(1 - F_{p,n}(r))^{N-n}dr$$

$$\Psi^{-i}(s) = p\int_0^s e^{-\lambda r(N-n)}(1 - F_{p,n}(r))^{N-n-1}d(-e^{-\lambda r}(1 - F_{p,n}(r))) \\ +(1-p)\int_0^s (1 - F_{p,n}(r))^{N-n-1}dF_{p,n}(r).$$

It follows from Lemma 1 that, for all j, Ψ^j is absolutely continuous on $[0, \Delta)$, i.e., $\Psi^j(s) = \int_0^s \psi^j(r)dr$. where ψ^i and ψ^{-i} are given by the following:

$$\psi^i(r) = p\lambda e^{-\lambda r(N-n+1)}(1 - F_{p,n}(r))^{N-n}$$

$$\psi^{-i}(s) = pe^{-\lambda s(N-n+1)}(\lambda + F'_{p,n}(s+) - \lambda F_{p,n}(s))(1 - F_{p,n}(s))^{N-n-1} + (1-p)(1 - F_{p,n}(s))^{N-n-1}F'_{p,n}(s+).$$

Substituting the expressions for both $V_{p,n}(\delta_0)$ and $V_{p,n}(\delta_\Delta)$ into (12) and rearranging, we obtain that for all $\Delta \in (0, \varepsilon)$,

$$K_1(\Delta) + K_2(\Delta) + K_3(\Delta) = 0 \tag{13}$$

where

$$K_1(\Delta) \equiv (\int_0^\Delta k_n[(\alpha_n(p(s)) - \alpha_n(p)) + \beta(1-p)]\psi^i(s)ds)/\Delta$$

$$K_2(\Delta) \equiv ((N-n)\int_0^\Delta [V_{p^{-i}(s),n+1} - k_n\alpha_n(p) + \beta(1-p)]\psi^{-i}(s)ds)/\Delta$$

$$K_3(\Delta) \equiv ((1 - \sum_j \lim_{s \to \Delta-} \Psi^j(\Delta))[k_n(\alpha_n(p(\Delta)) - \alpha_n(p)) + \beta(p(\Delta) - p)])/\Delta.$$

Now, we consider $\lim_{\Delta \to 0+}$ of $K_1(\Delta)$, $K_2(\Delta)$, and $K_3(\Delta)$ separately. For $K_1(\Delta)$, it follows from L'Hôpital's Rule, together with the continuity of $\alpha_n(p(\Delta))$ (Lemma 3) and $\psi^i(\Delta)$ in Δ that

$$\lim_{\Delta \to 0+} K_1(\Delta) = \beta(1-p)\psi^i(0) = \beta(1-p)p\lambda.$$

For $K_2(\Delta)$, it again follows from L'Hôpital's Rule that

$$\lim_{\Delta \to 0+} K_2(\Delta) = (N-n)[V_{p^{-i},n+1} - k_n\alpha_n(p) + \beta(1-p)](\frac{\lambda p}{\alpha_n(p)}).$$

For $K_3(\Delta)$, by the continuous differentiability of $\Psi^j(s)$ that $\lim_{\Delta \to 0+} \sum_j \lim_{s \to \Delta-} \Psi^j(s) = 0$. Thus, it follows from the right-differentiability of $\alpha_n(p(\Delta))$ in Δ that

$$\lim_{\Delta \to 0+} K_3(\Delta) = p'(\Delta)\Big|_{\Delta = 0+}[k_n\alpha_n'(p) + \beta] = -\lambda p(N-n+1)(1-p)[k_n\alpha_n'(p) + \beta].$$

Since we have shown that $\lim_{\Delta \to 0+} K_1(\Delta)$, $\lim_{\Delta \to 0+} K_2(\Delta)$, and $\lim_{\Delta \to 0+} K_3(\Delta)$ exist, and are given by the above expressions, it follows from (13) that

$$\lim_{\Delta \to 0+} K_1(\Delta) + \lim_{\Delta \to 0+} K_2(\Delta) + \lim_{\Delta \to 0+} K_3(\Delta) = 0.$$

Substituting the above expressions for $K_1(\Delta)$, $K_2(\Delta)$ and $K_3(\Delta)$, we obtain (ODE).

Now, we wish to establish that (ODE) must hold whenever $k_n \geq \beta$ or $p > p_n^*$. It follows from Proposition 1 that $\alpha_n(p) < 1$, and thus by the above, (ODE) holds.

Finally, we establish the two limit conditions presented in the proposition. We begin by establishing that when $k_n \geq \beta$, $\lim_{p \to 0+} \alpha_n(p) = \beta/k_n$. To this end, first note by Lemma 2 that for all $p > 0$, $V_{p,n}(\delta_0) = V_{p,n}(\delta_\infty)$. Note further that $\lim_{p \to 0+} V_{p,n}(\delta_\infty) = 0$. Thus, $\lim_{p \to 0+} V_{p,n}(\delta_0) = \lim_{p \to 0+} k_n\alpha_n(p) - \beta = 0$, and therefore, $\lim_{p \to 0+} \alpha_n(p) = \frac{\beta}{k_n}$. Next, let us consider the case where $k_n < \beta$. That $\lim_{p \to p_n^*+} \alpha_n(p) = 1$ follows from Lemma 3, since by Proposition 1, $\alpha_n(p_n^*) = 1$. □

I now I define a problem (P) on α. I first show that α constitutes an equilibrium if and only if it satisfies (P) and (SC) is optimal for a firm who has privately confirmed the state (Lemma 4). I then show that assuming α satisfies (P), it is indeed optimal for a firm who has confirmed the state to abide by (SC) (Lemma 5). The proofs of Lemma 4 and Lemma 5 are relegated to the Supplemental Appendix. Thus, existence and uniqueness of an equilibrium (Theorem 1) will reduce to establishing a unique solution to (P).

Definition 2. α is a solution to (P) if it satisfies the following for all n and $p \in (0,1]$:

1. If $k_n < \beta$ and $p \leq p_n^* \equiv \frac{k_n-\beta}{k_n/n-\beta}$, then $\alpha_n(p) = 1$.
2. If $k_n \geq \beta$ or $p < p_n^*$, then α satisfies (ODE), with limit condition $\lim_{p\to 0+} \alpha_n(p) = \beta/k_n$ if $k_n \geq \beta$ and $\lim_{p\to p_n*+} \alpha_n(p) = 1$ if $k_n < \beta$.
3. $\alpha_n(1) = 0$.

Lemma 4. *(α, F) is an equilibrium if and only if at all (p, n) on-path, α is both consistent with F and a solution to (P).*

Lemma 5. *Suppose that α is a solution to (P). Then, $F_{1,n}(0) = 1$ is optimal for all n.*

Proof of Theorem 1. Fix an n. Assume by induction that there exists a unique solution to (P) for all $m > n$. We wish to show that there exists a unique solution to (P) for n. It suffices to show there exists a unique solution to the following two problems, when $\beta \leq k_n$ and $\beta > k_n$, respectively:

$$\text{(ODE}'\text{) is satisfied on } [0,1)\text{, and } \alpha_n(0) = \beta/k_n \qquad \text{(BVP: } \beta \leq k_n\text{)}$$

$$\text{(ODE}'\text{) is satisfied on } (0,p_n^*]\text{, and } \alpha_n(p^*) = 1. \qquad \text{(BVP: } \beta > k_n\text{)}$$

where

$$\alpha_n'(p) = -\frac{1}{k_n(1-p)\alpha_n(p)}\frac{N-n}{N-n+1}[k_n\alpha_n(p) - \tilde{V}_{p^i,n+1} - \beta(1-\alpha_n(p))(1-p)], \quad \text{(ODE}'\text{)}$$

and

$$\tilde{V}_{p^i,n+1} = \begin{cases} V_{p^i,n+1} & \text{if } p^i \in (0,1) \\ 0 & \text{if } p^i \geq 1. \end{cases}$$

We do this by invoking the Picard existence and uniqueness theorem, and thus begin by establishing that the right-hand side of (ODE′) is Lipschitz continuous in $\alpha_n(p)$ and continuous in p for $p \in [-\varepsilon, 1)$ and $\alpha_n(p) \in [c, 1+\varepsilon]$ for any $c > 0$ and some $\varepsilon > 0$. Since $p^i \equiv \alpha_n(p) + (1-\alpha_n(p))p$, it suffices to show that $\tilde{V}_{p^i,n+1}$ is Lipschitz continuous in p^i for $p^i \geq 0$. In the case where $n = N$, $\tilde{V}_{p^i,n+1} = 0$ for all p^i, and this is immediate. Next, suppose $n > 1$. First, consider the case where $k_{n+1} \geq \beta$. It follows from Lemma 2 that:

$$\tilde{V}_{p^i,n+1} = \begin{cases} k_n \alpha_{n+1}(p^i) - \beta(1-p^i) & \text{if } p^i < 1 \\ 0 & \text{if } p^i \geq 1. \end{cases}$$

Because $\tilde{V}_{p^i,n+1}$ is continuously differentiable in p^i when $p^i \neq 1$, to establish that it is Lipschitz continuous it suffices to show that $\lim_{p^i \to 1-} V_{p^i,n+1} = 0$. Suppose this does not hold, by contradiction. Because $\alpha_{n+1}(\cdot)$ satisfies (ODE), this implies that $\lim_{p^i \to 1-} \alpha'_{n+1}(p^i) = \infty$. This implies that $\lim_{p^i \to 1} \alpha_{n+1}(p^i) = \infty$, and thus that (ODE) is not satisfied at $p^i = 1$. Contradiction. Next, consider the case where $k_{n+1} < \beta$:

$$\tilde{V}_{p^i,n+1} = \begin{cases} k_{n+1} p^i/(N-n) & \text{if } p^i < p^*_{n+1} \\ k_n \alpha_{n+1}(p^i) - \beta(1-p^i) & \text{if } p^i \in (p^*_{n+1}, 1) \\ 0 & \text{if } p^i = 1. \end{cases}$$

By the same reasoning as above, $\tilde{V}_{p^i,n+1}$ is Lipschitz continuous for all $p^i > p^*_{n+1}$. Furthermore, Lipschitz continuity holds for $p^i < p^*_{n+1}$. To show that Lipschitz continuity holds for all p^i, it suffices to show $\tilde{V}_{\cdot,n+1}$ is differentiable at p^*_{n+1}. To this end, we take the left- and right- derivative of $\tilde{V}_{\cdot,n+1}$ at p^*_{n+1} and show they are equal:

$$\frac{d}{dp}\tilde{V}_{p^*_{n+1}-,n+1} = \frac{k_{n+1}}{N-n} = \frac{d}{dp}\tilde{V}_{p^*_{n+1}+,n+1}.$$

Now, we show that there exists a unique solution for both (BVP: $\beta \leq k_n$) and (BVP: $\beta > k_n$) in some neighborhood of their respective boundary conditions. By the Picard Theorem, this follows immediately from our above-established result that the right-hand side of (ODE′) is Lipschitz continuous in $\alpha_n(p)$ and continuous in p in some neighborhood of the boundary conditions $(\alpha_n(p) = 1, p = p^*_n)$ and $(\alpha_n(p) = \beta/k_n, p = 0)$.

Next, we seek to establish global existence and uniqueness of solutions to both (BVP: $\beta \leq k_n$) and (BVP: $\beta > k_n$). First, consider (BVP: $\beta > k_n$). The argument for (BVP: $\beta \leq k_n$) follows analogously. Let $[p^*, \overline{p})$ denote the largest right-open interval such that existence and uniqueness are both satisfied. Assume by contradiction that $\overline{p} < 1$. Let $\alpha_n(p)$ denote the solution along this interval.

We begin by showing that on this interval, $\alpha_n(p) \in (\underline{\alpha}, 1]$, where $\underline{\alpha} > 0$ is some constant. The upper bound is established as follows: suppose by contradiction that $\alpha_n(p) > 1$ somewhere on the interval. By the continuous differentiability of α_n along the interval, there must exist some $q < p$ such that $\alpha_n(q) = 1$ and $\alpha_n'(q) \geq 0$. However, it follows from (ODE′) that $\alpha_n'(q) = -\frac{1}{k_n(1-q)}\frac{N-n}{N-n+1}[k_n - \tilde{V}_{p^i,n+1}] < 0$, where the strict inequality follows from the fact that $\tilde{V}_{p^i,n+1} < k_{n+1} \leq k_n$. Contradiction. The lower bound is established as follows: suppose by contradiction that such a lower bound does not exist. Then, again by the continuous differentiability of α_n along the interval, there exists some $\hat{p} \in [p_n^*, \overline{p})$ such that $\lim_{p \to \hat{p}-} \alpha_n(p) = 0$ and $\alpha_n(p) > 0$ for all $p < \hat{p}$. However, it then follows from (ODE) that $\lim_{p \to \hat{p}-} \alpha_n'(p) = \infty$. Thus, (ODE′) is not satisfied on $[p_n^*, \overline{p})$. Contradiction.

Having established that on $[p^*, \overline{p})$, $1 \geq \alpha_n(p) > \underline{\alpha} > 0$, it follows from (ODE′), and the observation that $\tilde{V}_{p^i,n+1}$ is bounded, that α_n' is also bounded on this range. Thus, it follows that $\lim_{p \to \overline{p}-} \alpha_n(p) \equiv \overline{\alpha} > 0$ exists.

Now, consider the following modified boundary value problem, which is identical to (BVP: $\beta > k_n$), except with boundary condition $(\overline{p}, \overline{\alpha})$. Recall we have shown that (ODE′) is Lipschitz continuous in $\alpha_n(p)$ and continuous in p in some neighborhood of $(\overline{p}, \overline{\alpha})$. Thus, we can again apply the Picard Theorem to obtain that there exists a unique solution to the modified boundary value problem in some neighborhood of $(\overline{p}, \overline{\alpha})$. I.e., there exists some $\varepsilon > 0$ such that there is a unique solution $\tilde{\alpha}_n(p)$ on interval $(\overline{p} - \varepsilon, \overline{p} + \varepsilon)$. We now "paste" this solution $\tilde{\alpha}_n$ with α_n. Let

$$\hat{\alpha}_n(p) = \begin{cases} \alpha_n(p) & \text{if } p \in [p_n^*, \overline{p}) \\ \tilde{\alpha}_n(p) & \text{if } p \in [\overline{p}, \overline{p} + \varepsilon). \end{cases}$$

$\hat{\alpha}_n(p)$ is a unique solution to (BVP: $\beta > k_n$) on $[p_n^*, \overline{p} + \varepsilon)$, which contradicts our earlier assumption that $[p^*, \overline{p})$ was the largest right-open interval such that existence and uniqueness are satisfied. □

Proof of Proposition 3 and Proposition 4. Let us begin by showing that $\alpha_n(p)$ is weakly decreasing in p for all (p, n) on-path. By Lemma 4, it follows that when $k_N < \beta$, $\alpha_N(p) = 1$ for all p, and when $k_N \geq \beta$, $\alpha_N'(p) = 0$ for all p. Thus, $\alpha_N(p)$ is constant in p. Now, consider the case where $n < N$. Assume by induction that $\alpha_{n+1}(p)$ is weakly decreasing in p whenever $(p, n+1)$ is on path.

Assume by contradiction that there exists some $\overline{p}$ such that α_n is strictly increasing. By Lemma 4, $\alpha_n'(p) = 0$ whenever $\beta \geq k_n$ and $p \geq p_n^*$. Thus it must be that $\beta < k_n$ or $\overline{p} > p_n^*$. In this case, (ODE) must be satisfied. Now define the function $X(p)$ as follows:

$$X(p) \equiv k_n \alpha_n(p) - \beta(1 - p^i) - V_{p^i,n+1}. \tag{14}$$

Whenever (ODE) is satisfied, the following holds:

$$\alpha_n'(p) > (=) 0 \text{ if and only if } X(p) < (=)\, 0. \tag{15}$$

Thus, $X(\overline{p}) < 0$. Now, I claim that there exists $\underline{p} < \overline{p}$ such that $\lim_{p \to \underline{p}+} X(p) \geq 0$. To establish this, first suppose $k_n \geq \beta$. In this case,

$$\lim_{p \to 0+} X(p) = (k_n + \beta) \lim_{p \to 0+} \alpha_n(p) - \beta - \lim_{p \to 0+} V_{\alpha_n(p),n+1}. \tag{16}$$

When $\lim_{p \to 0+} \alpha_{n+1}(\alpha_n(p)) < 1$, it follows from Lemma 2 that

$$\lim_{p \to 0+} V_{\alpha_n(p),n+1} = k_{n+1} \alpha_{n+1}(\beta/k_n) - \beta(1 - \beta/k_n).$$

Substituting this into (16), we obtain $\lim_{p \to 0+} X(p) = \beta - k_{n+1}\alpha_{n+1}(\beta/k_n)$. In the case where $k_{n+1} < \beta$, it follows directly that $\lim_{p \to 0+} X(p) \geq 0$. Otherwise, if $k_{n+1} \geq \beta$, then because $\lim_{p \to 0+} \alpha_{n+1}(p) = \beta/k_{n+1}$, it follows from the inductive assumption that $\alpha_{n+1}(p) \leq \beta/k_{n+1}$ for all p, and thus that $\lim_{p \to 0+} X(p) \geq 0$.

When $\lim_{p \to 0+} \alpha_{n+1}(\alpha_n(p)) = 1$, it follows from the inductive assumption that $\alpha_{n+1}(q) = 1$ for all $q \geq \lim_{p \to 0+} \alpha_n(p)$. Thus,

$$\lim_{p \to 0+} V_{p^i,n+1} = \lim_{p \to 0+} V_{p^i,n+1}(\delta_\infty) = \frac{k_{n+1}}{N - n} \frac{\beta}{k_n}.$$

Substituting into the above expression for $\lim_{p \to 0+} V_{p^i,n+1}$, we obtain $\lim_{p \to 0+} X(p) \geq 0$, implying by Lemma 4 that $k_{n+1} \geq \beta$.

Next, consider the case where $k_n < \beta$. In this case, $\lim_{p \to p_n^*+} X(p) = k_n - \lim_{p^i \to 1-} V_{p^i,n+1}$. If $\lim_{p^i \to 1-} \alpha_{n+1}(p^i) < 1$, then by Lemma 2,

$$\lim_{p^i \to 1-} V_{p^i,n+1} = \lim_{p^i \to 1-} V_{p^i,n+1}(\delta_0) = k_{n+1} \lim_{p^i \to 1-} \alpha_{n+1}(p^i) < k_n.$$

Thus, we obtain that $\lim_{p \to p_n^*+} X(p) > 0$. Meanwhile, if $\lim_{p^i \to 1-} \alpha_{n+1}(p^i) = 1$, by the inductive assumption, $\alpha_{n+1}(p) = 1$ for all p. Thus,

$$\lim_{p^i \to 1-} V_{p^i,n+1} = \lim_{p^i \to 1-} V_{p^i,n+1}(\delta_\infty) = \frac{k_{n+1}}{N-n}.$$

So in this case as well, $\lim_{p \to p_n^*+} X(p) > 0$. We have thus shown that there always exists $\underline{p} < \overline{p}$ such that $\lim_{p \to \underline{p}+} X(p) \geq 0$.

Because $X(\overline{p}) < 0$, there must exist some $q \in [\underline{p}, \overline{p}]$ such that $X(q) < 0$ and $X'(q) < 0$. Differentiating X, we obtain

$$X'(q) = k_n \alpha_n'(q) + \beta((1-q)\alpha_n'(q) + (1 - \alpha_n(q))) - \frac{d}{dq} V_{q^i,n+1}. \tag{17}$$

First, consider the case where $\alpha_{n+1}(q^i) < 1$. By Lemma 2,

$$V_{q^i,n+1} = V_{q^i,n+1}(\delta_0) = k_{n+1}\alpha_{n+1}(q^i) - \beta(1-q^i). \tag{18}$$

Substituting this into (17), we obtain $X'(q) = k_n \alpha_n'(q) - k_{n+1}\alpha_{n+1}'(q^i)((1-q)\alpha_n'(q) + (1-\alpha_n(q)))$. Because $X(q) < 0$, it follows from (15) that $\alpha_n'(q) > 0$. Furthermore, by the inductive assumption, $\alpha_{n+1}'(q^i) \leq 0$. Thus, $X'(q) > 0$. Contradiction.

Next, consider the case where $\alpha_{n+1}(q^i) = 1$. By the inductive assumption, $\alpha_{n+1}(p) = 1$ for all $p \leq q^i$. Thus, $V_{q^i,n+1} = V_{q^i,n+1}(\delta_\infty) = \frac{k_{n+1}q^i}{N-n}$. Substituting into (17):

$$X'(q) = k_n \alpha_n'(q) + (\beta - \frac{k_{n+1}}{N-n})((1-q)\alpha_n'(q) + (1-\alpha_n(q))). \tag{19}$$

Because $\alpha_{n+1}(q^i) = 1$, by Lemma 4, $\beta \geq k_{n+1}$. Thus, $X'(q) > 0$. Contradiction.

Next, we show that if $k_N \geq \beta$, then $\alpha_n(p) = \beta / k_n$. Assume that $k_N \geq \beta$. First consider $n = N$. By Lemma 4, $\alpha_n'(p) = 0$ for all p on-path, and thus, $\alpha_N(p)$ is constant in p. Since Lemma 4 also states that $\lim_{p \to 0+} k_N \alpha_N(p) = \beta$, it must be that $\alpha_N(p) = \beta / k_N$ for all p. Now, consider $n < N$. Assume by induction that $\alpha_{n+1}(p) = \beta / k_{n+1}$ for all p. We begin by showing that $\alpha_n(p)$ is constant in p. Since $k_n \geq \beta$, by Lemma 4, (ODE) must hold at all p. By (15), showing $\alpha_n(p)$ is constant in p is equivalent to showing that $X(p) = 0$. To establish this, I begin by claiming that $V_{p^i,n+1} = V_{p^i,n+1}(\delta_0)$. In the case where $k_{n+1} > \beta$, it follows from Proposition 1 that $\alpha_{n+1}(p^i) < 1$, and thus this follows from Lemma 2. In the case where $k_{n+1} = \beta$, $k_m = \beta$ for all $m \geq n+1$, and by Proposition 1, $\alpha_m(p) = 1$ for all p. Thus,

$V_{p,n+1}(\delta_s) = p\beta$. for all $\delta \in [0,\infty]$ and all p. Thus, $V_{p^i,n+1} = V_{p^i,n+1}(\delta_0)$. Having established that $V_{p^i,n+1} = V_{p^i,n+1}(\delta_0)$, we have: $V_{p^i,n+1} = k_{n+1}\alpha_{n+1}(p^i) - \beta(1-p^i) = \beta p^i$. Substituting this into (14), we obtain $X(p) = k_n\alpha_n(p) - \beta$. Since $\alpha_n(p)$ is weakly decreasing, $\alpha_n(p) \le k_n/\beta$ for all p, and thus $X(p) \le 0$. Separately, by (15) $\alpha_n(p)$ weakly decreasing implies that $X(p) \ge 0$. Combining these inequalities, we have $X(p) = 0$.

Finally, I show that $k_N < \beta$ implies that $\alpha_n'(p) < 0$ whenever $\alpha_n(p) < 1$. Suppose $k_N < \beta$, and suppose by contradiction that at some q such that $\alpha_n(q) < 1$, $\alpha_n'(q) = 0$. It follows from (15) that $X(q) = 0$. First, suppose $\alpha_{n+1}(q^i) = 1$. Recall from (19) that $X'(q) = (\beta - \frac{k_{n+1}}{N-n})(1-\alpha_n(q))$. Now, I claim that $\beta > \frac{k_{n+1}}{N-n}$. When $n = N-1$, this follows from the assumption that $k_N < \beta$. When $n < N-1$, because $\alpha_{n+1}(q^i) = 1$, this is a result of Proposition 1. Thus, $X'(q) > 0$. Since $X(q) = 0$ for some $p < q$, $X(p) < 0$. By (17), $\alpha_n'(p) > 0$. This contradicts $\alpha_n(p)$ being weakly decreasing in p. Next, suppose $\alpha_{n+1}(q^i) < 1$. By (18), $X'(q) = -k_{n+1}\alpha_{n+1}'(q)[1-\alpha_n(q)] > 0$. This implies that there exists some $p < q$ such that $X(p) < 0$ and thus that $\alpha'(p) > 0$. Contradiction. □

Proof of Proposition 5. I begin by showing that $C_n(p)$ is strictly decreasing in k_n whenever $b_n(p) > 0$. Fix parameters $\{k\}_{m=n+1}^N, \beta$ and λ. Consider $k_n > \hat{k}_n \ge k_{n+1}$. Let α ($\hat{\alpha}$), b ($\hat{b}$) and C ($\hat{C}$) denote equilibrium objects under k_n ($\hat{k}_n$), respectively. We want to show that for any p such that $\hat{b}_n(p) > 0$, $C_n(p) < \hat{C}_n(p)$.

First I claim that for all p such that $\hat{b}_n(p) > 0$, $b_n(p) > \hat{b}_n(p)$. Suppose not, by contradiction. Then there exists a q such that $b_n(q) \le \hat{b}_n(q)$ and $\hat{b}_n(q) > 0$. By Proposition 2, there exists a $0 < \overline{p} < q$ such that

$$b_n(p) > \hat{b}_n(p) \text{ for all } p \in (p_n^*, \overline{p}] \text{ and } \hat{b}_n(p) = b_n(p) = 1 \text{ for all } p < p_n^*,$$

where

$$p_n^* = \begin{cases} \frac{k_n - \beta}{\frac{k_n}{N-n+1} - \beta} & \text{if } \beta > k_n \\ 0 & \text{if } \beta \le k_n. \end{cases}$$

Thus, by the continuity of $b_n(p)$, there exists a p^* such that

$$b_n(p^*) = \hat{b}_n(p^*) \text{ and } b_n(p) > \hat{b}_n(p) \text{ for all } p \in (p_n^*, p^*).$$

By the differentiability of α_n: $|\hat{\alpha}_n'(p^*)| \ge |\alpha_n'(p^*)|$.However it follows from (ODE) that since $\hat{\alpha}_n(p^*) = \alpha_n(p^*)$, $|\hat{\alpha}'_n(p^*)| < |\alpha'_n(p^*)|$. Contradiction. Now, I claim that for all p such that $\hat{b}_n(p) > 0$, $b_{n+1}(p(1-\alpha_n(p)) + \alpha_n(p)) < \hat{b}_{n+1}(p(1-\hat{\alpha}_n(p)) + \hat{\alpha}_n(p))$. First note that $b_{n+1}(p) = \hat{b}_{n+1}(p)$ for all p. Since $b_{n+1}(p)$ is strictly increasing in p and $\alpha_n(p) < \hat{\alpha}_n(p)$, the statement follows. Since we have shown that for all p such that $\hat{b}_n(p) > 0$, $b_{n+1}(p(1-\alpha_n(p))+\alpha_n(p)) < \hat{b}_{n+1}(p(1-\hat{\alpha}_n(p))+\hat{\alpha}_n(p))$ and $b_n(p) > \hat{b}_n(p)$, it follows that $C_n(p) < \hat{C}_n(p)$.

Now, I prove the second part of the claim. Fix a p and parameters $\{k\}_{m=n+1}^N, \beta$ and λ. Since we have shown that $C_n(p)$ is strictly increasing in k_n when $b_n(p) > 0$, to prove the statement it suffices to show that (1) there exists a $k_n^h > k_{n+1}$ such that $C_n(p) < 0$ under k_n^h and (2) $C_n(p) > 0$ under $k_n = k_{n+1}$. I begin by showing (1). It follows from Proposition 2 that for any $k_n > \beta$, $\alpha_n(p) \le \frac{\beta}{k_n}$ for all p. Thus, for any $k_n > \beta$

$$b_n(p) \ge \lambda p(\frac{k_n}{\beta} - 1) \text{ and } b_{n+1}(\tilde{p}) \le b_{n+1}((\frac{\beta}{k_n})(1-p) + p).$$

So, for any k_n,

$$C_n(p) \le b_{n+1}(\frac{\beta}{k_n}(1-p) + p) - \lambda p[\frac{k_n}{\beta} - 1] \equiv L(p, k_n).$$

Now, note that

$$\lim_{k_n \to \infty} L(p, k_n) = b_{n+1}(p) - \lim_{k_n \to \infty} \lambda p[\frac{k_n}{\beta} - 1] = -\infty.$$

Thus, there exists a k_n^h such that $C_n(p) < 0$. Next, I show that $C_n(p) > 0$ when $k_n = k_{n+1}$. Suppose by contradiction that $k_n = k_{n+1}$ and at some p where $b_n(p) > 0$, $b_{n+1}(\tilde{p}) \le b_n(p)$. Since $\tilde{p} > p$, this implies $\alpha_{n+1}(\tilde{p}) > \alpha_n(p)$. So,

$$k_n\alpha_n(p) - V_{\tilde{p},n+1} - \beta(1-\alpha_n(p))(1-p) \le k_n\alpha_n(p) - k_{n+1}\alpha_{n+1}(\tilde{p}) < 0.$$

Thus, by (ODE), $\frac{d}{dt}\alpha_n(p(t)) > 0$, contradicting Proposition 3 and Proposition 4.

□

Proof of Corollary 3. It suffices to show that $\lim_{p\to 0+} b_{n+1}(\tilde{p}) - b_n(p) > 0$. It follows from Proposition 2 and (4) that $\lim_{p\to 0+} b_n(p) = 0$. Also, $\lim_{p\to 0+} \tilde{p} = \lim_{p\to 0+} \alpha_n(p) = \beta/k_n$, where the final equality follows from Proposition 2. Thus, $\lim_{p\to 0+} b_{n+1}(\tilde{p}) = b_{n+1}(\lim_{p\to 0+} \tilde{p}) = b_{n+1}(\beta/k_n)$, Thus,

$\lim_{p \to 0+}[b_{n+1}(\tilde{p}) - b_n(p)] = b_{n+1}(\beta/k_n) > 0.$ □

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