Competition and Herding in Breaking News

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Abstract

I present a dynamic model of breaking news. Firms are rewarded for preempting their competitors and for making credible reports. Errors occur when firms fake, reporting without evidence. While even monopolists err, competition and observational learning exacerbate errors and give rise to rich dynamics in reporting. Competition intensifies faking by engendering a preemptive motive, but is endogenously mitigated by improvement in credibility over time. Observational learning causes errors to propagate through the market via a *copycat effect*, where a new report triggers a surge in faking. The copycat effect causes herding on the timing of news.

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1. Introduction

> *What a newspaper needs in its news, in its headlines, and on its editorial page is terseness, humor, descriptive power, satire, originality, [...] and accuracy, accuracy, accuracy!*
>
> — Joseph Pulitzer

Accuracy is often considered the core tenet of news media. This belief is widely held by consumers of news: the majority of US survey respondents listed accuracy as a primary function of news, valuing it over thorough coverage, unbiasedness, and relevance (Pew Research Center, 2019).

Despite this, public perceptions of accuracy are not favorable: 35% of respondents state that news organizations do a good job reporting the news accurately (Pew Research Center, 2022). While many factors may contribute to this skepticism, consumers express particular concern about hasty reporting: 53% of respondents state that news breaking too quickly is a major reason for errors.

These concerns are justified by many instances of factual errors by news media. In the aftermath of the 9/11 attacks, cable news stations made several statements that were false: NBC News reported an explosion outside the pentagon, CNN reported a fire outside the national mall, and CBS News claimed the existence of a car bomb outside the state department.[1] Erroneous reporting has been endemic to terrorist attacks in general, with news media misidentifying perpetrators or other key details of the Boston bombings, Sandy Hook massacre, and London bombings. Furthermore, such errors are not limited to terrorist attacks. In 2004, CBS News published the Killian Documents, a collection of memos which called into question George W. Bush's military record. These documents could never be authenticated and were widely believed to be forged. In 2017, ABC News falsely reported that Michael Flynn would testify that Donald Trump had directed him "to make contact with the Russians."[2]

While such errors are commonplace, they are also costly to news firms. For one, exposure of errors can be reputationally damaging. This was especially true of the *Rolling Stone* scandal, in which the magazine falsely accused a group of University of Virginia students of sexual assault. Not only was the journalistic failure widely reported, it resulted in several publicized lawsuits against the magazine. Furthermore, errors can lead firms to oust journalists in an apparent effort to protect their reputations. This was evident in the terminations of Dan Rather and Brian Ross

[1] https://www.reuters.com/article/idUS182595581320121217
[2] https://www.nytimes.com/2017/12/02/us/brian-ross-suspended-abc.html

—both lead journalists at major news stations—following their respective reporting blunders.

The objective of this paper is twofold. First, I seek to understand why reporting errors are pervasive despite their costliness to firms. In particular, I consider how the strategic and learning environment news firms face can induce them to commit errors that are avoidable. My second objective is to study the dynamics of breaking news. Namely, I ask when over the course of a news cycle firms are less credible and more prone to erring.

To answer these questions, I present a dynamic model of breaking news. Firms learn privately about a story by receiving confirmation that it is true, and choose if and when to report it. Errors occur when firms fake, i.e., report despite lacking confirmation. Because reports are public, firms also learn by observing the reporting behavior of opponents. Regarding payoffs, firms are penalized for errors but rewarded for market share, which depends on two qualities of the report. First, all else equal, a firm who preempts its rivals enjoys greater market share. Second, market share depends on the credibility of the firm's report, which is the consumers' belief that the story was confirmed before being reported. Namely, a report is consumed only to the extent that there is trust in the firm's journalistic standards.

I establish existence and uniqueness of an equilibrium. Under this equilibrium, the firm randomizes across faking times: fake reports are made as if they are being generated by a non-homogenous Poisson process. The indifference condition that supports this mixing implies an ordinary differential equation (ODE) on the arrival rate of fake reports, and the equilibrium is characterized by a recursive system of these ODEs, a fact that is central to the analysis.

In equilibrium, errors are responses to three features of the breaking news environment: a lack of commitment by firms, competition, and observational learning. Competition and observational learning not only exacerbate faking, they induce dynamics in firm behavior. I begin by showing that errors can occur even in the absence of competition: if the cost of error is relatively small, even a monopolist will fake. Such errors are driven by a firm's inability to commit to a reporting strategy. Because consumers cannot detect faking, credibility is unaffected by deviations in the firm's reporting strategy. Firms are thus tempted to fake in order to capitalize on their credibility after it has been established. I substantiate this intuition by showing that a monopolist that could commit to a reporting strategy would never fake.

I then analyze a multi-firm setting, and find that both competition and observational learning can exacerbate errors, and do so in different ways. Competition incentivizes speed by giving rise to a preemptive motive, which makes faking more valuable. To restore indifference, credibility must fall to make faking less valuable, which in equilibrium can only be consistent with more faking. Notably, this preemptive motive is not merely an artifact of the firm's payoff function, but rather an equilibrium phenomenon. In particular, a preemptive motive arises in equilibrium when the cost of error is relatively large, but when the cost of error is sufficiently small, credibility endogenously adjusts in such a way that the preemptive motive disappears. Meanwhile, observational learning causes existing errors to propagate through the market. This is because, like consumers, firms cannot detect faking. Thus, an erroneous report by one firm makes other firms more confident that the story is true. This in turn implies a lower risk of error and a greater threat of preemption — because it is more likely that another firm will privately confirm the story — both of which yield the firm more inclined to fake.

Reporting dynamics take two different forms: gradual changes in the absence of a new report and discrete changes in response to a new report. In the absence of a new report, firms gradually become more truthful, i.e., less inclined to fake. Furthermore, whenever there is a preemptive motive, firms become gradually more credible in the eyes of consumers. In other words, consumers are more skeptical of quick reports, a finding which conforms with documented concerns about hasty news reporting. The reason for this gradual improvement in credibility lies in the firms incentives. The risk of preemption introduces an endogenous cost to delay, and the firm must somehow be compensated for this cost to ensure that its indifference condition is satisfied. This is achieved by means of increasing credibility. That is, credibility increases to mitigate the haste-inducing effects of preemption enough to yield the firm indifferent between faking and waiting.

Meanwhile, a report by a rival firm causes a discrete change in a firm's reporting behavior. This can take the form of a *copycat effect*, in which one firm's report causes an instantaneous and persistent boost in faking. The copycat effect is always intensified by observational learning, and is precisely the channel through which observational learning propagates errors. When the copycat effect occurs, firms not only herd on the decision to report, but also the timing of their reports. Furthermore, this herding on report timing applies not only to errors, but valid stories as well. In addition to anecdotal evidence of clustering in the timing of news errors [3], such herding has been

[3] Examples include the reporting errors surrounding the Boston bombings (https://www.nytimes.com/ 2013/04/18/business/media/fbi-criticizes-false-reports-of-a-bombing-arrest.html) and the 2000 US presidential election (https://www.nytimes.com/2000/11/08/us/the-2000-elections-the-media-a-

documented in the empirical literature. Cagé, Hervé, and Viaud (2020) find that in 25% of cases, a news story is reported by a different media outlet within 4 minutes of being published by the original news breaker. I provide a rationale for such herding that is grounded in the strategic and learning environment news media face.

Finally, I consider comparative statics with respect to the cost of error, the speed of learning, and the number of firms in the market. A higher cost of error improves credibility by making faking less profitable, while faster learning curbs faking by yielding firms more quickly pessimistic about the story's truth. Meanwhile, firm entry has a more nuanced effect on news quality. It can deteriorate credibility early on in the news cycle by increasing competition, and thus the preemptive motive firms face. However, this deterioration is mitigated — and potentially reversed — later on in the news cycle by the entering firm's positive effect on the market's ability to learn observationally that the story is false.

Related Literature This paper adds to the literature on games of preemption. In the classic application to technology adoption (Reinganum (1981), Fudenberg and Tirole (1985)) firms benefit from waiting for the cost of a technology to fall before adoption, but benefit less from adoption if they are preempted. In these papers, as is standard throughout the literature, the benefit of delay and cost of preemption are exogenous. I depart from this by considering a setting where both this benefit and cost are endogenous. I find that even when there is no exogenous benefit to delay, it arises as an equilibrium phenomenon: delay is beneficial whenever preemption is costly. Furthermore, firms may be endogenously rewarded for succeeding their rivals in such a way that nullifies preemptive motives. In other words, even if there is an exogenous cost of preemption, it can be completely mitigated in equilibrium.

I contribute more specifically to the literature on observational learning in preemption games. In Hopenhayn and Squintani (2011) and Bobtcheff, Levy, and Mariotti (2022), players learn about their opponents' propensity to act by observing how long they last without doing so. I instead consider a setting where players learn observationally about a variable of common value, i.e., where there are learning externalities. Such has been studied by Moscarini and Squintani (2010), Bobtcheff et al. (2022), and Chen, Ishida, and Mukherjee (2023). Both Moscarini and Squintani (2010) and Bobtcheff et al. (2022) study winner-takes-all research races with good and bad news learning, respectively. Despite considering a winner-takes-all setting, Moscarini and Squintani (2010) document herding in the timing of actions, wherein

flawed-call-adds-to-high-drama.html).

players quit the race simultaneously. Chen et al. (2023) study a market entry game that is not winner-takes-all, finding that entry by one firm can trigger another firm to immediately follow suit.

Herding often arises in games with endogenously-timed decisions with learning externalities but without payoff externalities (Chamley and Gale (1994), Grenadier (1999), Murto and Välimäki (2011)). Chamley and Gale (1994) and Grenadier (1999) study investment timing games, showing that endogenously-timed information cascades, where one player's action triggers others to immediately follow suit, can occur. Meanwhile, Murto and Välimäki (2011) document dynamics that are qualitatively highly similar to the equilibrium of this paper: players exit the game with a time-varying hazard rate that rises when an opponent exits. That is, herding is not immediate and deterministic but rather gradual and probabilistic. In both these settings, gradual herding occurs because information cascades would make delay strictly profitable. In Murto and Välimäki (2011), this is due to learning externalities: observing how opponents react to an exit allows a player to make a more informed exit decision. In my setting, it is due to the endogenous payoff function: deterministic reports carry no credibility, and thus are not profitable. Furthermore, because the model I present entails payoff externalities in addition to learning externalities, reverse herding is also possible. I.e., firms sometimes fake with a lower probability following an opponent report.

In application, this paper contributes to a literature on competition in news, as surveyed by Gentzkow and Shapiro (2008). More recently, Chen and Suen (2023) and Galperti and Trevino (2020) consider the effects of competition on news accuracy when firms face costs or constraints to accuracy. Meanwhile, I consider a setting where accuracy is not intrinsically costly but rather entails the strategic cost of being preempted. Such is studied by papers on preemption in news (Lin (2014), Pant and Trombetta (2019), Andreottola and de Moragas (2020)). As in this paper, Lin (2014) models a setting where firms dynamically learn about a story and decide whether and when to report it. I further incorporate two key elements of breaking news in my model: credibility and observational learning. Together these two features drive the qualitative features of equilibrium, including reporting dynamics and herding. I contribute more generally to this literature by studying the effects of competition on the dynamics of news. Namely, I show that competition can give rise to dynamics in reporting behavior that are otherwise absent.

Finally, the notion of faking arises in other work. In Boleslavsky and Taylor (2024), a single agent decides whether to fake a project or wait for a valid one, and a principal decides whether to approve it. They similarly find that the agent generates fakes via a non-homogenous Poisson process, but that faking increases in the short run. Their dynamics are driven by the principal's increasing optimism that the agent is ethical, and thus unwilling to fake, as time passes. The Poisson arrival of inaccurate information is also a feature of Che and Hörner (2018) and entails spamming by recommender systems.

The remainder of the paper is organized as follows. Section 2 presents the model. In Section 3, I characterize the equilibrium and the role of commitment. In Section 4, I present the core economic implications of this equilibrium, which pertain to competition and dynamics. I present comparative statics in Section 5. Section 6 concludes. All formal proofs are relegated to the Appendix.

2. A model of breaking news

There are $N \geq 1$ firms, indexed by i, and one consumer. Time, which is continuous and has an infinite horizon, is denoted by $t \in [0, \infty)$. There exists some story, and the time-invariant state $\theta \in \{0, 1\}$ denotes whether it is true ($\theta = 1$) or false ($\theta = 0$). At $t = 0$, all players are endowed with a common prior $p_0 \equiv Pr(\theta = 1) \in (0, 1)$.

Learning and reporting Firms learn about θ via one-sided Poisson signals: if $\theta = 1$, a private signal revealing that $\theta = 1$ arrives to each firm at a Poisson rate $\lambda > 0$, where the time of this arrival is independent across firms.[4] Formally, letting $s_i \in [0, \infty]$ denote the time at which such a conclusive signal arrives to firm i, with $s_i = \infty$ denoting that a signal never arrives, $s_i \sim (1 - e^{-\lambda s_i})$ if $\theta = 1$, and $s_i = \infty$ if $\theta = 0$. This learning process approximates a setting where firms pursue reliable sources that can confirm a story, rather than seeking piecemeal evidence. In the case of a terrorist attack, this could entail reaching out to contacts at the police department.

Firms choose whether and when to report the story. Specifically, at any t, a firm can choose to make a report as long as they have not already done so. As the payoff function will illustrate, the content of this report can be interpreted as an assertion that the story is true, i.e., that $\theta = 1$. A report history H is a partially ordered set of pairs (i, t_i), pairing each firm i who has reported with a report time t_i, with elements ordered according to the order in which the reports were made.[5] Report histories are public: all players observe the current report history.

[4] An extension where firms have heterogeneous learning abilities is included in the Online Appendix.

[5] Formally, elements are ordered according to relation $\succsim$, where $(i, t_i) \succ (j, t_j)$ if $t_i > t_j$ or $t_i = t_j$ but i reported first, and $(i, t_i) \sim (j, t_j)$ if the reports were made simultaneously.

Payoffs A firm who never reports earns a payoff of 0. A firm who does report earns

$$k_n\alpha - \beta\mathbb{I}[\theta = 0]. \tag{1}$$

The first term ($k_n\alpha$) is the market share (i.e., viewership or readership) that the firm enjoys from reporting a story. It is the product of k_n, a parameter capturing the role of the firm's order n, and α, the credibility of the report. More precisely, the index n denotes that the firm was the n^{th} to report: $n \equiv |H| + 1$, where H denotes the current history at the time of the report. I assume that $k_1 \geq k_2 \geq ... \geq k_N \geq 0$, i.e., firms who report earlier than their competitors earn greater market share, all else equal. Meanwhile, credibility α denotes the consumer's belief, at the time that the report is made, that the firm has independently confirmed the state. Formally, it is the belief that $s_i \in [0, t]$, where t is the time of the report. In assuming a product form for market share, I take the stance that consumers value accuracy in journalism, and thus only consume news to the extent that they find it credible. [6] The second term of (1), $-\beta\mathbb{I}[\theta = 0]$, is the penalty of error: a firm who reports when $\theta = 0$ incurs a penalty $\beta > 0$. This captures the reputational harm a firm suffers from making a report that is later uncovered to be false.

Equilibrium A Markov[7] strategy F is a set of distributions $F_{p,n}$ over future report times for each belief $p \equiv Pr(\theta = 1)$ and order n of the next firm to report. [8] Specifically, the span of time the firm waits before reporting, conditional on not receiving a conclusive signal, is distributed according to $F_{p,n} \in \Delta[0, \infty]$ where ∞ denotes a lack of report. I restrict attention to symmetric equilibria, and thus omit the firm's index in much of the analysis.

I place some restrictions on F. First, I assume that for all (p, n), $F_{p,n}$ is piecewise twice differentiable and right-differentiable everywhere on $[0, \infty)$. This grants analytical convenience and ensures that equilibrium objects are well-defined. Second, I impose a selection criterion (SC): a firm immediately reports a story it knows is true. This is stated as Definition 1.

Definition 1. F satisfies (SC) if $F_{1,n}(0) = 1$ for all $n \in \{1, ..., N\}$.

This criterion rules out equilibria with periods of silence supported by pessimistic

[6] A microfoundation for this formula for market share is presented in the Online Appendix.

[7] In general, the Markov state should also include the identities of the remaining firms and beliefs over their private signals. However, it is without loss to define the state in this way within the class of strategy profiles that satisfy the below criteria, namely symmetry and (SC).

[8] If multiple firms report at the same history H, one firm will be assigned order n, another $n + 1$, etc., with their identities randomly determined according to a uniform distribution.

off-path beliefs, i.e., beliefs that reports made during these gaps have little or no credibility. In making this assumption, I am selecting an equilibrium rather than imposing restrictions on firms' behavior, namely, it is optimal for firms to abide by (SC) in equilibrium. Conveniently, (SC) implies that fixing an n and starting belief p, all remaining players hold the same common belief about the state after t time has passed, assuming no new reports are made. This common belief is denoted by $p(t)$, and it follows from Bayes Rule that:

$$p(t) = \frac{pe^{-\lambda(N-n+1)t}}{pe^{-\lambda(N-n+1)t} + (1-p)}. \tag{2}$$

Defining strategies in this way, i.e. with a separate distribution for each (p, n), is convenient but introduces redundancy. Specifically, for any (p, n) and $t > 0$, $F_{p,n}$ and $F_{p(t),n}$ "overlap": both distributions specify the firm's reporting behavior at $(p(t+s), n)$ for any $s \geq 0$. Thus, I impose that the $F_{p,n}$ must be mutually consistent[9]: at any (p, n) on-path and $t > 0$,

$$F_{p(t),n}(s) = \frac{F_{p,n}(t+s) - F_{p,n}(t_-)}{1 - F_{p,n}(t_-)} \text{ for all } s \geq 0 \text{ whenever } F_{p,n}(t) < 1, \tag{3}$$

where $F_{p,n}(t_-) \equiv \lim_{\tau \uparrow t} F_{p,n}(\tau)$. Let $\mathcal{F}$ denote set of distributions F that satisfy the above restrictions.

Before proceeding, I define two terms to describe reporting: faking and truth telling. A report at time t is *fake* if it is made despite the firm lacking independent confirmation, i.e., a signal $s^i \notin [0, t]$. Meanwhile, a report that is made after the firm has confirmation is *truthful*. Under the selection assumption (SC), strategies differ only in their distributions over fake reports.

I seek a symmetric Markov perfect equilibrium of this game. This is a Markov strategy F paired with beliefs α and p at each history such that F is sequentially rational and the beliefs are consistent with Bayes Rule. The consistency of α with Bayes Rule implies the following at all (p, n) on-path: [10]

$$\alpha_n(p) = \begin{cases} \frac{\lambda p}{\lambda p + b_n(p)} & \text{if } F_{p,n}(0) = 0 \\ 0 & \text{if } F_{p,n}(0) > 0, \end{cases} \tag{4}$$

[9] This condition is analogous to the closed-loop property specified in Fudenberg and Tirole (1985). I adopt the term *consistency condition* from Laraki, Solan, and Vieille (2005), who define this condition for a general class of continuous-time games of timing.

[10] This formula is derived by applying Bayes Rule to a discrete-time approximation of the beliefs that obtain under this game. This derivation is presented in the Online Appendix.

where $b_n(p) \equiv F'_{p,n}(0+)$, the right-derivative of $F_{p,n}$ at 0, is the instantaneous arrival rate of fake reports. This formula is intuitive. If $F_{p,n}(0) > 0$, there is a point mass of fake reports at (p, n), and because conclusive signals are distributed continuously over time, the instantaneous probability of a truthful report is zero. So, the consumer and all competing firms are certain that a report made at (p, n) was fake, and thus assign to it zero credibility. If there is not a point mass of fake reports at (p, n), credibility is assessed by comparing the arrival rates of truthful reports (λp) to that of fake reports ($b_n(p)$), assigning higher credibility to reports made when the arrival rate of fake reports is relatively low.

3. Equilibrium characterization

This section presents the equilibrium characterization. I begin by defining the firm's problem and establishing two properties that are instrumental to the analysis. Then, as a stepping stone to the full model characterization, I consider the monopoly case. This elucidates the forces at play even when competition is absent. Finally, I characterize the equilibrium of the full model under competition.

3.1. The firm's problem

I now present the firm's problem. I begin by defining a useful object, the *first report distribution*. Fix a report history H and strategy profile F, and let p denote the common belief and n the order of the next firm to report. Index the firms who have not yet reported by i. The first report distribution $\Psi^i(s)$ denotes the probability that player i reported when or before s time has passed and was not preempted by any of the remaining firms (i.e., i was the first to make a new report). This is given by:

$$\Psi^i(s) = p \int_0^s e^{-\lambda r(N-n)} \prod_{j \neq i} (1 - F^j_{p,n}(r)) d(e^{-\lambda r}(F^i_{p,n}(r) - 1)) + (1-p) \int_0^s \prod_{j \neq i} (1 - F^j_{p,n}(r)) dF^i_{p,n}(r).$$

The firm's value from playing strategy F^i at (p, n) given each of its opponents plays F^j can then be written recursively as

$$V_{p,n}(F^i) = \int_0^\infty [k_n \alpha_n(p(s)) - \beta(1 - p^i(s))] d\Psi^i(s) + \sum_{j \neq i} \int_0^\infty V_{p^j(s), n+1}(F^i) d\Psi^j(s), \quad (5)$$

where $V_{p,N+1} \equiv 0$ and $p^j(s)$ denotes the common belief when s time has passed, conditional on no new reports having been made, except for a report by j at time s. The first integral of (5) is firm i's expected payoff from reporting conditional on being the first of the remaining firms to do so, and the second integral is conditional on being preempted. Specifically, upon being preempted by j at time s, the state changes discretely from $(p(s), n)$ to $(p^j(s), n+1)$. Thus, the firm's continuation value upon being preempted is its value at this new state.

The firm's problem at (p, n) is

$$\max_{F^i \in \mathcal{F}} V_{p,n}(F^i).$$

3.2. Properties of equilibrium

I now present two necessary conditions on a firm's equilibrium strategy. First, I show that there cannot exist any jumps (i.e., point masses) in the distribution of fake reports. Second, whenever a firm is less-than-fully credible, it must satisfy certain indifference conditions. Similar properties arise in other games with continuous strategy spaces, where they result from competition.[11] However, as I will illustrate below, here they are instead driven by the endogenous nature of credibility and thus hold even without competition.

First, let us consider the "no jumps" property (Lemma 1):

Lemma 1. *In equilibrium, at any (p, n) on-path, $F_{p,n}$ is continuous everywhere when $p < 1$.*

To see why this holds, recall that a report made when there is a point mass of faking yields zero credibility. Meanwhile, faking while also not being certain that the story is true yields a strictly positive expected penalty $\beta(1-p)$. Thus, a firm's value from faking at such a time is strictly negative. The firm could then profitably deviate by truth telling: this would preclude the firm from making an error, ensuring a weakly positive payoff.

Now let us state the indifference property. To this end, let δ_s for $s \in [0, \infty]$ denote the distribution that places full mass on faking when s time has passed. Specifically, δ_0 denotes immediate faking, while δ_∞ denotes that the firm never fakes.

Lemma 2. *In equilibrium if $\alpha_n(p) < 1$ and (p, n) is on-path, there exists an $\varepsilon > 0$ such that*

$$V_{p,n} = V_{p,n}(\delta_s) \text{ for all } s \in [0, \varepsilon) \cup \infty,$$

where $V_{p,n}(\delta_s)$ is the value from playing δ_s at (p, n) and F at (q, m) for all q and $m > n$.

Lemma 2 states that whenever $\alpha_n(p) < 1$, the firm must find a number of strategies optimal. First, it must be optimal to fake immediately (play δ_0). Second, it must be

[11] These include war of attrition games (Hendricks, Weiss, and Wilson (1988)) and all-pay auctions (Baye, Kovenock, and De Vries (1996)).

optimal to be truthful for some sufficiently short span of time dt and then fake (play δ_{dt}). Third, it must be optimal to never fake (play δ_∞). I will now provide some insight into the proof. Let us begin by considering why δ_{dt} must be optimal for $dt \in [0, \varepsilon]$. It follows from our regularity conditions on the firm's strategy that $\alpha_n(p(s))$ must be right-continuous in s. So, if $\alpha_n(p) < 1$, $\alpha_n(p(s)) < 1$ for all s sufficiently small. Furthermore, whenever $\alpha_n(p(s)) \in (0, 1)$, the firm is faking with a strictly positive hazard rate. This means that the firm mixes between faking with delay $[0, \varepsilon]$, implying that all such pure strategies are optimal. Next, let us consider why never faking must be optimal. If it were not, then a firm who has not received a conclusive signal must fake with probability 1. To achieve this, the firm must sustain a sufficiently high hazard rate of faking as t tends to ∞. But because the hazard rate of truthful reports tends to zero as p falls, credibility would tend to zero, making faking suboptimal.

3.3. The monopoly characterization and the role of commitment

I now characterize the equilibrium under a monopoly, i.e., assuming $N = 1$. As there is only one firm, I drop the n index from all functions and parameters.

Claim 1. *Under a monopoly, for all p on-path:* $\alpha(p) = \min\{\beta/k, 1\}$.[12]

Claim 1 states that the monopolist's credibility is constant over time and not always perfect. In particular, credibility is less-than-perfect whenever $\beta/k < 1$. That is, errors can occur when the ex-post penalty of error is relatively low. In the remainder of this subsection, I provide intuition for these properties and show that the monopolist's errors are driven by its inability to commit to a reporting strategy.

Let us first consider why credibility is constant. Lemma 2 established that whenever $\alpha(p) < 1$, the firm must be indifferent between faking immediately and after some wait dt. By the martingale property of firm's belief p, both of these strategies yield the same expected penalty from error $\beta(1 - p)$. So, for both strategies to be optimal, they must also yield the same expected market share $k\alpha$. Thus, credibility must be constant. It is noteworthy that this reasoning is predicated on the fact that waiting is costless. Indeed, this is true under a monopoly: not only is waiting intrinsically costless (there is no discounting), a monopolist does not incur the strategic cost to waiting that preemption entails. As I show in Section 4, this strategic cost of waiting is precisely what gives rises to dynamics in credibility under competition.

[12] A formal proof of this claim is omitted, as it is a special case of the N-period characterization that follows (Proposition 1 and Proposition 2).

Though a monopolist's credibility is constant, its strategy is dynamic: the hazard rate of faking (b) strictly decreases and tends to zero whenever credibility is less than one. This follows from (4) and the firm's one-sided Poisson learning process: the absence of a report means that the firm has not received a conclusive signal, causing the common belief to drift down. For credibility to remain constant, the hazard rate of faking must decline as well.

Now, let us consider why truth-telling cannot be sustained when $\beta/k < 1$, and why credibility is equal to β/k. Suppose by contradiction that $\beta/k < 1$ and the firm is truthful. This implies full credibility, and thus that the market share ($k\alpha(p)$) exceeds the penalty of error (β). So, it is strictly optimal to report, even if the story is false. That is, the firm can profitably deviate by faking. We conclude that in any equilibrium, the firm fakes with positive probability. To pin down the level of credibility, we recall from Lemma 2 that a firm who fakes must be indifferent between faking immediately and remaining truthful. Indeed, there is a unique value of credibility that ensures indifference: β/k. There is some intuition behind this: the bigger β/k is, the more costly errors are compared to market share for any α, and thus the more costly faking is relative to truth telling. So, α must correspondingly increase to maintain indifference.

In this model, I assume that consumers cannot detect faking and firms cannot commit to a reporting strategy. Rather, a firm optimizes its strategy, for instance by faking, taking the credibility function as given. I show that allowing the monopolist to commit, it would always be truthful and thus never make errors. Formally, I consider a modified version of the model where the firm announces, and commits to, a reporting strategy before the consumer assesses credibility, presented formally in the Appendix. Claim 2 states that in this setting, a monopolist never fakes.

Claim 2. *Under commitment, the unique monopolist equilibrium is such that $b(p) = 0$ for all p on-path.*

One can immediately see that given the ability to commit, the firm would always choose truth telling over its non-commitment equilibrium strategy, even when $\beta < k$. By committing to truth telling, the firm is guaranteed a payoff of k if $\theta = 1$, and 0 if $\theta = 0$. Meanwhile, under the no-commitment equilibrium, the firm will earn strictly less when $\theta = 1$, due to its strictly lower credibility, and earn 0 when $\theta = 0$ because the market share from reporting exactly offsets the penalty of error. Intuitively, committing to truth telling is better for the firm because the enhanced credibility the firm enjoys when the story is true exceeds any payoff it might enjoy from reporting a false story (which is zero in equilibrium). Indeed, truth telling is not only superior to the non-commitment solution, it is the unique commitment solution. This result illustrates that faking not only deteriorates the quality of information consumers receive, it also harms firms. Despite this, faking occurs because firms cannot credibly promise truthfulness to consumers.

3.4. Full model characterization

Now, I characterize the equilibrium of the full model (i.e., under N firms). I show that any equilibrium is the solution to a recursive set of boundary value problems.

Let us begin by deriving the conditions under which the firm is truthful. This both serves as a stepping stone to a full characterization and illustrates how competition can deteriorate credibility and exacerbate faking. This result is stated as Proposition 1.

Proposition 1. *In equilibrium, at any (p, n) on-path, $\alpha_n(p) = 1$ if and only if:*

1. $k_n \leq \beta$
2. $p \leq p_n^* \equiv \min\{\frac{k_n - \beta}{\frac{k_n}{N-n+1} - \beta}, 1\}$.

Proposition 1 provides two conditions that are necessary and sufficient for truth telling. The first condition alone, $k_n \leq \beta$, was sufficient for truth telling under a monopoly (Claim 1). However, under competition, a second condition is required: the common belief must lie below some threshold p_n^*.

The need for this additional condition illustrates that truth telling is harder to sustain under competition. This is due to the fact that under competition, truth-telling entails a risk of being preempted. Assuming that being preempted is costly, it follows immediately that truth-telling is harder to sustain. But we cannot take for granted that being preempted is costly. It is, however, true that preemption is costly conditional on being truthful in equilibrium. This is most obvious in a *winner takes all* setting, where $k_n = 0$ for all $n > 1$. In this case, the costliness of being preempted is an artifact of the parameters, as a preempted firm can earn at best zero payoff. But in general, the decreasing nature of k_n alone does not imply that preemption is costly: improved credibility for succeeding firms could endogenously counteract the decay in k_n, making preemption costless or even valuable. Indeed, I will show in the next section that under certain parameters, precisely such a phenomenon occurs in equilibrium. But conditional on a firm being truthful, this cannot happen: truthfulness implies full credibility, leaving no room for a succeeding firm to improve on it.

Let us now consider why under competition, truthtelling is only possible when firms are sufficiently pessimistic about the story. This can be explained by the fact that faking and truth telling each pose a different kind of risk to the firm: while truth telling entails the risk of being preempted, faking entails the risk of making an error. Both of these depend on the belief p about the state: higher p implies a lower probability of error and a higher probability of being preempted. The former is immediate, and the latter is due to the fact that preemption is more likely when the story is true. Namely, conditional on the story being true, an opponent reports not just because it is faking, but also because it has received confirmation. Because a lower risk of error and higher risk of preemption both make faking relatively more profitable, truth telling is harder to sustain when p is high.

It remains to characterize the firm's behavior when truth telling does not hold. To this end, I obtain a key result: when the firm fakes, credibility must satisfy an ODE and limit condition.

Proposition 2. *In equilibrium, at all (p, n) on-path where $k_n \geq \beta$ or $p > p_n^*$, the following ODE must be satisfied:*

$$\alpha_n'(p) = -\frac{1}{k_n(1-p)\alpha_n(p)} \frac{N-n}{N-n+1}[k_n\alpha_n(p) - V_{\tilde{p},n+1} - \beta(1-\alpha_n(p))(1-p)], \quad \text{(ODE)}$$

where $\tilde{p} \equiv \alpha_n(p) + (1-\alpha_n(p))p$.

Furthermore, $\lim_{p\to 0+} \alpha_n(p) = \beta/k_n$ if $k_n > \beta$ and $\lim_{p\to p_n^*+} \alpha_n(p) = 1$ if $k_n \leq \beta$.

The proof for Proposition 2 follows from the indifference condition established in Lemma 2. When credibility is less than 1, there exists an $\varepsilon > 0$ such that the strategies δ_Δ yield the same payoff for all $\Delta \in (0, \varepsilon]$. This implies

$$\frac{d}{d\Delta+} V_{p,n}(\delta_\Delta)\bigg|_{\Delta=0} = 0. \tag{6}$$

It follows from (5) that

$$V_{p,n}(\delta_\Delta) = \int_0^\Delta k_n\alpha_n(p(s))d\Psi^i(s) + (N-n)\int_0^\Delta V_{p^{-i}(s),n+1}d\Psi^{-i}(s) + \\ (1 - \sum_j \lim_{s\to\Delta-} \Psi^j(s))[k_n\alpha_n(p(\Delta)) - \beta(1-p(\Delta))],$$

where Ψ is the first-report distribution associated with the strategy profile in which i plays δ_∞ and all $j \neq i$ play the equilibrium strategy $F_{p,n}$. Differentiating, we obtain

$$\lim_{\Delta\to 0+} \frac{d}{d\Delta} V_{p,n}(\delta_\Delta) = [\frac{dp}{dt}(k_n\alpha_n'(p)) - \frac{\lambda p(N-n)}{\alpha_n(p)}(V_{\tilde{p},n} - V_{\tilde{p},n+1})]. \tag{7}$$

Setting the right-hand side to zero, in accordance with (6), yields (ODE). Equation (7) illustrates that waiting to fake, rather than faking immediately, has two consequences for the firm's payoff. The first is that α_n, and thus the market share from reporting, may change. This rate of change is $\frac{dp}{dt}(k_n \alpha_n'(p))$. The second consequence is that the firm risks being preempted: this happens at a Poisson rate $\frac{\lambda p(N-n)}{\alpha_n(p)}$, in which case its expected payoff changes by $V_{\tilde{p},n} - V_{\tilde{p},n+1}$. I call this decrease in value the *regret* from preemption.

Let us examine the rate and regret of preemption more closely. As one might expect, the rate of preemption is increasing in the number of rival firms remaining $(N - n)$ and the expected rate at which these rivals can confirm the story (λp). It is also decreasing in credibility: less credible firms are more likely to fake, and thus more likely to preempt. Meanwhile, the regret of preemption is the difference between two values, $V_{\tilde{p},n+1}$ and $V_{\tilde{p},n}$. $V_{\tilde{p},n+1}$ denotes the firm's continuation value in the event that it is preempted at (p, n). This value is taken at $(\tilde{p}, n+1)$ because preemption affects both the firms order and the common belief: while the common belief was p prior to the rival firm's report, it increases to $\tilde{p} \equiv \alpha_n(p) + (1 - \alpha_n(p))p$ in its immediate aftermath. This expression for $\tilde{p}$ demonstrates that a rival firm's report means one of two things: either the report was triggered by a conclusive signal, in which case the new belief should be 1, or it was fake, in which case the new report offers no new information and the belief remains p. Since faking is unobservable, the new common belief $\tilde{p}$ is an average of these two conditional beliefs, where the weight given to the report being informed is its credibility. Meanwhile, $V_{\tilde{p},n}$ denotes the continuation value conditional on not being preempted. This value is not assessed at the belief prior to preemption p, but rather the posterior $\tilde{p}$. In this sense, $V_{\tilde{p},n+1} - V_{\tilde{p},n}$ denotes firm's regret from not having reported after being preempted.

In addition to (ODE), Proposition 2 establishes that one of two limit conditions must hold. Which condition holds depends on the model parameters, and like (ODE), these conditions result from the firm's indifference condition. First consider the case where $k_n \leq \beta$. It follows from Proposition 1 that $\alpha_n(p) = 1$ whenever $p \leq p_n^*$. Thus, $\alpha_n(p)$ limits to 1 as the belief approaches p_n^*. Otherwise, there would be an upward discontinuity at p_n^*, meaning that at beliefs close to p_n^*, the firm could profitably deviate by waiting until p_n^* to fake, causing a failure of indifference. When $k_n > \beta$, the firm never truth tells, so by Lemma 2 it must always be indifferent between faking and truth telling. As p approaches zero, a firm who fakes does so being nearly certain that it will incur penalty β. Thus, the payoff from faking limits to the following:

$$\lim_{p\to 0+} V_{p,n}(\delta_0) = k_n \lim_{p\to 0+} \alpha_n(p) - \beta.$$

Meanwhile, the value from truth-telling limits to zero, as it becomes certain that the firm never receives a conclusive signal and thus never reports. So, the limit condition in this case, $\lim_{p\to 0+} \alpha_n(p) = \beta/k_n$, ensures that indifference holds in the limit.

To take stock, Proposition 1 and Proposition 2 provide two necessary conditions on equilibrium credibility. They pin down the region in which truth telling occurs (Proposition 1), and show that otherwise, credibility must satisfy a recursive boundary value problem (Proposition 2). One can show that these two conditions are sufficient for an equilibrium as well, provided that the firm's strategy is consistent with this credibility function. Specifically, I show that if credibility satisfies these conditions, the firm cannot profitably deviate from the strategy that is consistent with this credibility. On the region where credibiilty is perfect, a deviation would consist of faking. Proposition 1 establishes that such a strategy cannot be played in equilibrium, that is, the firm could profitably deviate by truth telling even when their opponents are faking (the risk of being preempted is higher) and credibility is less-than-perfect (the benefit of reporting is lower). Such a strategy thus cannot be more profitable than truth telling when the firm's opponents are not faking, and credibility is perfect. On the region where $\alpha_n(p) < 1$, the firm's strategy involves mixing between faking and truth telling. This too must be optimal, because both (ODE) and the boundary conditions guarantee it.

Thus, the equilibrium is fully characterized by the solution to a recursive set of boundary value problems. While I do not derive a closed-form solution to this problem, I use the Picard-Lindelof theorem to establish existence and uniqueness. This result is stated as Theorem 1.

Theorem 1. *There is a unique equilibrium, where uniqueness applies at (p, n) on-path.*

4. Dynamics and herding

With the above characterization in hand, I study dynamics. I show that credibility gradually improves over time whenever preemption is costly, with discrete changes triggered by the report of a rival firm. Under certain conditions, in particular when observational learning is sufficiently strong, firms herd on their opponents' decisions to report as well as the timing of these reports. This is due to a *copycat effect*, wherein one report causes a surge in faking by others.

The nature of equilibrium dynamics hinges on whether the last firm fakes. Thus, I will discuss two separate cases: the first where the last firm is truthful ($k_N \leq \beta$) and the second where the last firm fakes with positive probability ($k_N > \beta$). I show that firms face a preemptive motive when the last firm is truthful, but this motive endogenously disappears otherwise. I begin by showing that credibility strictly improves over time when $\beta > k_N$, as long as no new reports are made.

Proposition 3. *If $\beta > k_N$, $\frac{d}{dt}\alpha_n(p(t)) > 0$ and $\frac{d}{dt}b_n(p(t)) < 0$ whenever $\alpha_n(p(t)) < 1$, for all (p, n) on-path.*

The broad implication of this result is that while credibility is constant under a monopoly, competition can give rise to dynamics. To understand why, it can be helpful to observe the following: as long as $\alpha_n(p(t))$ has not reached its upper bound of 1, it strictly increases if and only if there is a positive regret to preemption. Formally, this follows from (ODE). It is especially clear when we write (ODE) in the following form:

$$\frac{d}{dt}\alpha_n(p(t)) = \frac{\lambda p(t)(N-n)}{\alpha_n(p(t))k_n}[V_{\tilde{p},n} - V_{\tilde{p},n+1}]. \tag{8}$$

There is intuition behind this result. Whenever the firm is less-than-fully credible, it must be indifferent between faking immediately and waiting some length of time before doing so. However, if credibility remained constant, reporting immediately would be strictly better—it would allow the firm to avoid being preempted while suffering no harm to its credibility. To restore indifference, the firm must somehow be compensated for waiting. This can only be achieved by means of increasing credibility. That is, credibility must increase to mitigate the haste-inducing effects of preemption.

I have argued that credibility must increase when there is a positive regret from preemption, but as discussed above, this is not necessarily true even when there are multiple firms in the market. However, it is indeed true that preemption is costly when $\beta > k_N$, i.e. when β is high enough to ensure the last firm is truthful. The proof requires a backwards induction argument, but its core reasoning is most easily illustrated in a duopoly setting ($N = 2$) where $\beta \in (k_2, k_1)$. In this case, a firm fakes with a positive hazard rate as long as nobody has reported yet, but switches to truth telling as soon as its opponent makes a report. Proposition 3 asserts that the credibility of the first report α_1 must strictly increase over time. To see why, suppose instead that α_1 were constant, as in the monopoly case.[13] Since $k_1\alpha_1(p(t))$ must limit to β

[13] This argument is purely illustrative; it does not rule out the possibility that $\alpha_1(p(t))$ is increasing in t, nor that the function is only locally non-increasing. A formal treatment is presented in the proof.

(Proposition 3), it follows that $k_1\alpha_1(p(t)) = \beta$ for all t. This implies a failure of the firm's indifference condition: the market share from reporting first is so high that faking is strictly optimal. Specifically, if the story is false both faking and truth telling yield 0 payoff, but if the story is true the firm is ensured a payoff of β by faking but by truth telling risks being preempted and only earning k_2. So, the market share of the first firm must instead be strictly less than β and approach it from below. This restores indifference because it increases the value of truth-telling in two ways: (1) the lower market share from reporting first lowers the regret of being preempted and (2) increasing α provides an additional incentive to wait.

Proposition 3 also states that $b_n(p(t))$ is decreasing in t, an immediate corollary of the increasing nature of credibility. While this same result obtains in the monopoly case, the strictly increasing nature of credibility implies that $b_n(p(t))$ decays more quickly than under the monopoly equilibrium. I.e., the firm's preemptive motive also gives rise to more extreme dynamics in faking.

So far, we have restricted attention to the case where $k_N < \beta$. One can show that if this does not hold, preemption becomes costless in equilibrium and the dynamics in α_n disappear. I formalize this as Proposition 4.

Proposition 4. *If $k_N \geq \beta$, $\alpha_n(p) = \beta/k_n$ for all (p, n).*

This result states that when $k_N \geq \beta$, credibility is constant at a level where the market share $k_n\alpha_n(p)$ is not affected by the firm's order. That is, firms enjoy higher credibility from reporting after their opponents, which mitigates the decline in k_n in such a way that makes preemption costless. To understand why, it is again helpful to consider the duopoly case, but this time assuming that $\beta < k_2 < k_1$. In this case, the market share for the second reporter, $k_2\alpha_2(p)$, equals β no matter when that report is made. Now let us consider the first reporter. Again, the market share of the first reporter must limit to β, but cannot limit to β from below. If it did, a firm could profitably deviate by being truthful: being preempted would *benefit* the firm, as it would yield a higher market share β. Instead, the market share of the first firm, $k_1\alpha_1(p)$, must always equal β, which is exactly the market share for the second firm.

Proposition 3 and Proposition 4 also have implications for the effects of competition: they show that competition exacerbates faking. To formalize this, let $\bar{b}_n$ denote equilibrium faking under a monopoly ($N = 1$) with maximal market share k_n. This denotes equilibrium faking under a counterfactual where competition is absent. Corollary 1 states that under any state (p, n), faking is higher than under the competition-free counterfactual.

Corollary 1. *For any* (p, n), $b_n(p) \geq \bar{b}_n(p)$, *where the inequality holds strictly whenever* $b_n(p) > 0$ *and* $\beta \in (k_N, k_n)$.

To see why, note that Proposition 2 establishes that under competition, credibility limits to the value that obtains under a monopoly as $p \to 0$. Meanwhile, Proposition 3 and Proposition 4 establish that credibility is decreasing in p. This implies that credibility under competition lies below the monopoly value for all p, and thus, faking lies above the monopoly value.

Let us take stock of these results. Proposition 3 asserts that under certain conditions, news reports that are made with greater delay for research are more trustworthy to consumers. I.e., all else equal, consumers have greater trust in a firm's journalistic standards when a report is not made quickly. In this sense, this model provides a justification for consumer distrust of hasty reporting that originates from the firm's preemptive motive. Meanwhile, Proposition 4 establishes a notable feature of equilibrium: competition alone does not imply preemptive motives. Even though reporting first yields a higher market share all else equal, payoffs sometimes endogenously adjust in such a way that makes preemption costless. Because the firm's continuation value is determined inductively, whether a preemptive motive obtains in equilibrium hinges on the incentives of the last firm.

Proposition 3 and Proposition 4 describe the dynamics of reporting conditional on no new reports being made. For a more complete picture of equilibrium dynamics, it is helpful to plot simulations of credibility and faking over the course of time. Figure 1 does this for the case when $k_N < \beta$. By Proposition 3, credibility is increasing and faking decreasing as long as no new reports are made. However, new reports trigger discrete changes in credibility and faking, and these jumps are non-monotonic. Dynamics are qualitatively different when $k_N \geq \beta$ as illustrated by Figure 2. By Proposition 4, credibility is flat, with new reports triggering upwards jumps. But even in this case, a new report triggers an increase in faking.

Both these simulations illustrate the copycat effect, in which a new report triggers an increase in faking. I define it formally below.

Definition 2. A report at (p, n) triggers the *copycat effect* if $b_{n+1}(\tilde{p}) - b_n(p) > 0$.

Let us consider what forces are responsible for this effect. To this end, recall that a new report affects two changes to the state. First, it increments the order of the next firm to report from n to $n+1$. Second, firms learn observationally from the report, and thus the common belief increases from p to $\tilde{p}$. The following decomposition isolates the respective impacts of these two changes:

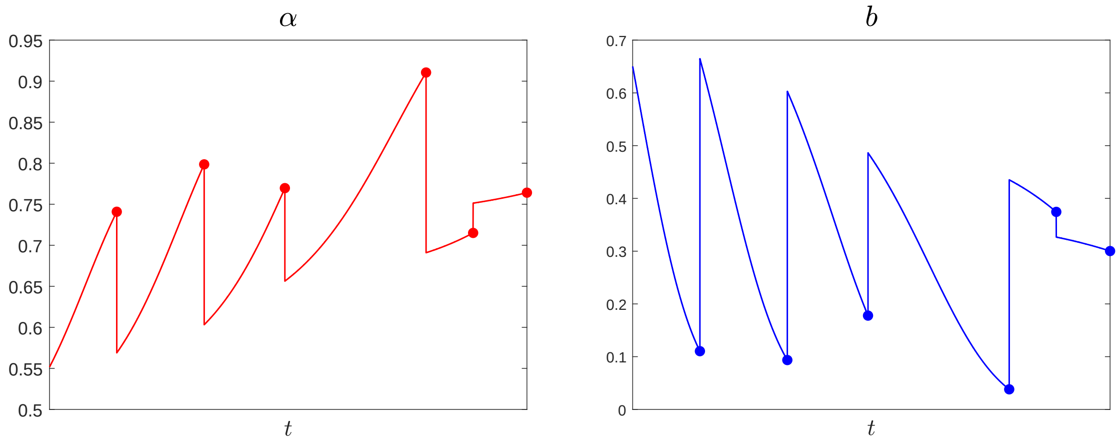

Figure 1: Simulation of credibility (α) and the hazard rate of faking (b), over the course of a game when $k_N < \beta$. Upwards jumps in b illustrate the copycat effect.

$$b_{n+1}(\tilde{p}) - b_n(p) = \underbrace{[b_{n+1}(p) - b_n(p)]}_{\text{change in order}} + \underbrace{[b_{n+1}(\tilde{p}) - b_{n+1}(p)]}_{\text{change in belief}}.$$

In equilibrium, the change in order has an ambiguous effect on faking, i.e., $b_{n+1}(p) - b_n(p)$ may be positive or negative. This is because a report can cause either an increase or decrease in the remaining firms' preemptive motive depending on the curvature of the k_n. To illustrate this, it is helpful to study two contrasting examples. First, consider a three firm setting ($N = 3$) where $k_1 > k_2 = k_3$ and $\beta \in (k_3, k_1)$. In this case, firms have a preemptive motive as long as nobody has yet reported, but this motive disappears once at least one firm has reported since the firm's order will no longer impact its market share. So here, a change in order reduces the incentive to fake: $b_2(p) - b_1(p) < 0$. Next, let us consider the same example but now assuming that $k_1 = k_2 > k_3$. In this case, all else equal, the first and second firm to report enjoy the same market share. So, firms face no cost to preemption as long as nobody has reported yet. Instead, this cost appears as soon as the first report is made. So in this case, a change in order increases the incentive to fake: $b_2(p) - b_1(p) > 0$.

Unlike the change in order, observational learning always causes faking to increase. This is stated as Corollary 2.

Corollary 2. *For any $n < N$, $b_{n+1}(\tilde{p}) - b_{n+1}(p) \geq 0$, where the inequality holds strictly whenever $b_{n+1}(\tilde{p}) > 0$.*

This is an immediate corollary of Proposition 3 and Proposition 4, which establish that

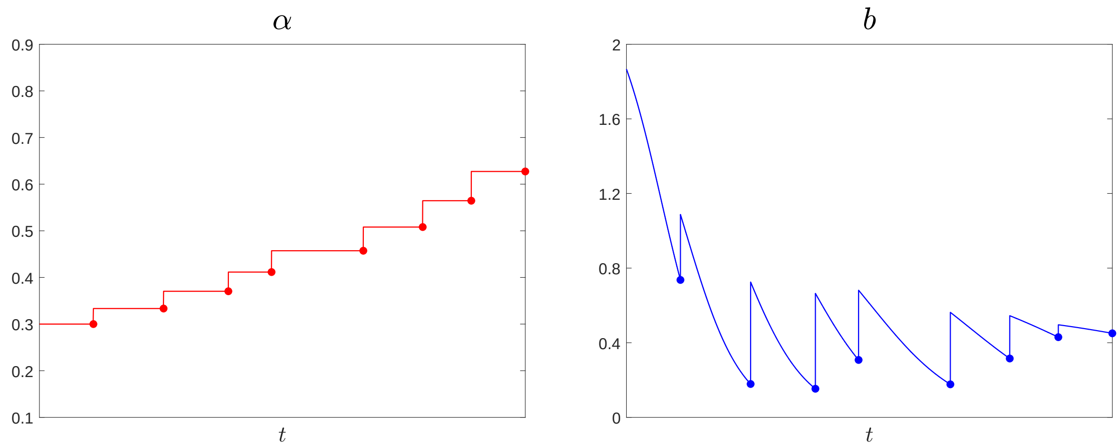

Figure 2: Simulation of credibility (α) and the hazard rate of faking (b), over the course of a game when $k_N > \beta$.

whenever a firm fakes, $b_n(p(t))$ is decreasing in t. Because the common belief $p(t)$ is decreasing in t, this means $b_n(p)$ is increasing in p. That is, an increase in the common belief implies an increase in faking. There is intuition for this as well: all else equal, a higher p implies a greater incentive to fake because this corresponds to a lower risk of error and higher risk of preemption, which both make faking relatively more valuable.

While observational learning will always cause faking to increase, the ambiguous effect of order means that the net effect is also ambiguous, i.e., a report does not always trigger the copycat effect. However, the copycat effect always occurs when the common belief is sufficiently low. I formalize this as Corollary 3.

Corollary 3. *Suppose $n < N$ and $k_{n+1} > \beta$. There exists a $\overline{p} > 0$ such that for all $p < \overline{p}$, $b_{n+1}(\tilde{p}) - b_n(p) > 0$.*

This result is driven by the fact that the magnitude of observational learning, $\tilde{p} - p$, is decreasing in the starting belief p. This is true for two reasons. First, a high pre-report belief p leaves little room for the belief to increase further. Second, reports made when p is high are less credible, and thus have less impact on the common belief. This negative correlation between the common belief and observational learning means that the positive effect of observational learning on faking is salient when p is low. Indeed, when p is sufficiently small, observational learning is substantial enough to give rise to the copycat effect.

The copycat effect has important implications for the behavior of firms: in the aftermath of an opponent report, a firm may be more likely to report not because they have also verified the story, but because they are faking. Meanwhile, in the absence of a competitor report, faking declines. That is, firms herd on the decision to report a story. Furthermore, because under the copycat effect firm faking increases immediately and then starts its gradual decline, a new report is most likely in the immediate aftermath of an opponent report. That is, firms also heard on the *timing* of reports, both correct and erroneous ones. Such herding in the timing of news is empirically documented by Cagé et al. (2020), and the copycat effect demonstrates that the interaction between strategic motives and social learning can explain such behavior.

5. Comparative statics

I now consider how the equilibrium changes with the cost of error (β), the rate of learning (λ) and the number of firms (N). I discuss each parameter in turn.

5.1. Cost of error (β)

Fixing a state (p, n), a firm is more credible and fakes less under a high β. This result is stated as Comparative Static 1.

Comparative Static 1. *In equilibrium, for any* (p, n)*,* $\alpha_n(p)$ *is weakly increasing in* β *and* $b_n(p)$ *is weakly decreasing in* β*, and strictly so whenever* $\alpha_n(p) < 1$*.*

This results from the firm's equilibrium incentives: a higher β makes faking more costly for the firm. Thus, increasing β will either induce the firm to resort to truth telling, or to restore indifference, require that it is compensated for this costlier faking with higher credibility. In either case, this implies lower credibility and higher faking. This result suggests that costlier errors can improve the quality of information news firms provide. Perhaps more surprisingly, a higher β can be beneficial for firms as well. Indeed, in a winner-takes-all setting, firms' ex-ante value in equilibrium is increasing in β. This is stated as Corollary 4.

Corollary 4. *In a winner-takes-all setting, a firm's ex-ante value,* $V_{p_0,1}$*, is increasing in* β *and strictly so whenever firms fake with positive probability.*

This result is connected to firms' inability to commit to truthful reporting. As discussed in Section 3.3, a monopolist who is able to commit to a reporting strategy will choose truth telling over any strategy that involves faking because truth telling ensures perfect credibility. A high β has a similar effect: it serves as a commitment device, curbing firm faking and thus improving credibility. Furthermore, this improvement in credibility is enough to outweigh the increased cost of error.

5.2. Speed of learning (λ)

I now consider the effect of an increase in the rate of learning (λ) on the equilibrium. A higher λ has no effect on credibility, and increases faking, under any state (p, n). However, increasing λ also speeds up the decay in the common belief, and thus credibility improves as a function of time.

Comparative Static 2. *In equilibrium, for any (p, n), $\alpha_n(p)$ is constant in λ, while $b_n(p)$ is weakly increasing in λ. Meanwhile, $\alpha_n(p(t))$ is increasing in λ for any (p, n) and $t > 0$.*

λ does not affect $\alpha_n(p)$ because it does not enter the boundary value problem which dictates the firm's credibility. However, an increase in λ does cause an increase in the arrival rate of valid reports (those triggered by the arrival of a conclusive signal). Thus faking, $b_n(p)$, increases to maintain a constant level of credibility. While λ does not affect $\alpha_n(p)$, changes in λ will affect the common belief $p(t)$: under a high λ, firms learn about the state more quickly and thus will hold a lower belief at any time $t > 0$ condition on not receiving a conclusive signal. This lower belief implies a higher expected cost of erring, which makes faking more costly. This must be counterbalanced by higher credibility at every $t > 0$ to ensure indifference holds.

5.3. Number of firms (N)

Finally, I study the effect of a change in the number of firms. This exercise sheds light on how firm entry can affect the quality of news. Comparative Static 3 establishes that firm entry deteriorates the credibility of the first report, and increases faking, but only if the report is made sufficiently early. In fact, market entry can improve credibility for reports made with sufficient delay.

Comparative Static 3. *In equilibrium, for any N, there exists a $\underline{t} > 0$ such that $\alpha_1(p_0(t))$ is weakly decreasing in N, and strictly so if $\alpha_1(p_0) < 1$, for all $t < \underline{t}$.*

This result can be understood by noting that an additional firm affects two separate changes to the market. First, each firm faces more competition, and thus greater preemption risk. Second, the market has a higher ability to learn observationally, and thus the common belief decays more quickly in the absence of a report. The effect of an additional firm can be understood as the combination of these two countervailing forces: higher competition, which deteriorates credibility, and a greater ability to learn, which per Comparative Static 2 improves credibility. An increase in learning ability has a negligible impact on credibility for early reports because firms learn gradually, and it thus takes time for differences in learning to substantially impact beliefs. However, an increase in competition will have a non-negligible impact on credibility even when $t = 0$. Thus, the impact of higher competition dominates when t is small, resulting in a net reduction in credibility. However, as time passes and the effect of faster learning grows, a reversal may take place, i.e., credibility may improve.

6. Conclusion

This paper presents a dynamic model of breaking news that accounts for the strategic and learning environment in the market for news. Firms face payoff and learning externalities, which exacerbate reporting errors and introduce dynamics in reporting behavior in distinct ways. The preemptive motive firms encounter causes errors by incentivizing hasty reporting and is responsible for lower news credibility that is gradually improving over time. Meanwhile, observational learning causes existing errors to propagate through the market via a copycat effect, where a report by one firm induces an immediate and persistent surge in faking by other firms, behavior that is consistent with clustering in the timing of news reporting. Crucially, this herding is not only driven by observational learning, but also by its interaction with the preemptive motive that firms face. Thus, more generally, this paper sheds light on how payoff and learning externalities can interact in a game of timing. To understand this interaction in more general payoff and learning environments is a topic that warrants further investigation. Beyond this, this paper illustrates how the core tradeoff in games of preemption—between the strategic benefit of preemption and the non-strategic benefit of delay—can arise as an equilibrium phenomenon in settings with endogenous payoff functions. Understanding more generally when such phenomena may occur in games of timing is another avenue of future work.

Appendix

Proof of Lemma 1. Let us begin by showing that at all (p,n) on-path such that $p<1$, $F_{p,n}$ is continuous at 0. To this end, suppose by contradiction that $F_{p,n}$ is discontinuous at 0. By the right-continuity of $F_{p,n}$, this implies that $F_{p,n}(0)>0$. Because (p,n) is on path, by (4), $\alpha_n(p)=0$. Furthermore, it follows by definition that $p^i(0)=p$. Recalling that we are restricting attention to symmetric equilibria, let Ψ denote the first-report distribution at (p,n) under the equilibrium strategy profile $F_{p,n}$. Because $F_{p,n}(0)>0$, $\Psi^i(0)>0$ for all i who have not yet reported.

Now define the following deviation $\hat{F}_{p,n}$. This strategy is identical to $F_{p,n}$, except that all the mass that $F_{p,n}$ places on 0 is shifted to ∞:

$$\hat{F}_{p,n}(s)=\begin{cases}F_{p,n}(s)-F_{p,n}(0) & \text{if } s<\infty\\ 1 & \text{if } s=\infty.\end{cases}$$

Now, fix some i who has not yet reported. Let $\hat{\Psi}$ denote the first-report distribution at (p,n) under the strategy profile where i plays $\hat{F}_{p,n}$ and all $j\neq i$ play $F_{p,n}$. By definition, for all $s\geq 0$, $\hat{\Psi}^i(s)=\Psi^i(s)-\Psi^i(0)$. Then,

$$\int_0^\infty [k_n\alpha_n(p(s))-\beta(1-p^i(s))]d\hat{\Psi}^i(s)>\int_0^\infty [k_n\alpha_n(p(s))-\beta(1-p^i(s))]d\Psi^i(s).$$

Again by definition, for all $s\geq 0$, $\hat{\Psi}^{-i}(s)=\Psi^{-i}(s)+X(s)$, where

$$\begin{aligned}X(s)\equiv \Psi^i(0)[p\int_0^s(1-F_{p,n})^{N-n-1}(1-\hat{F}_{p,n}(r))e^{-\lambda r(N-n)}d(e^{-\lambda r}(F_{p,n}(r)-1))\\ +(1-p)\int_0^s(1-F_{p,n}(r))^{N-n-1}(1-\hat{F}_{p,n}(r))dF_{p,n}(r)].\end{aligned}$$

Then, we have

$$\int_0^\infty V_{p^{-i}(s),n+1}d\hat{\Psi}^{-i}(s)-\int_0^\infty V_{p^{-i}(s),n+1}d\Psi^{-i}(s)=\int_0^\infty V_{p^{-i}(s),n+1}dX(s)\geq 0.$$

where the final inequality follows from $X(s)$ increasing in s and $V_{p^{-i}(s),n+1}\geq V_{p^{-i}(s),n+1}(\delta_\infty)\geq 0$. Combining the above two inequalities we have

$$\begin{aligned}V_{p,n}(\hat{F}_{p,n})=\int_0^\infty [k_n\alpha_n(p(s))-\beta(1-p^i(s))]d\hat{\Psi}^i(s)+(N-n)\int_0^\infty V_{p^{-i}(s),n+1}d\hat{\Psi}^{-i}(s)\\ >\int_0^\infty [k_n\alpha_n(p(s))-\beta(1-p^i(s))]d\Psi^i(s)+(N-n)\int_0^\infty V_{p^{-i}(s),n+1}d\Psi^{-i}(s)=V_{p,n}(F_{p,n}).\end{aligned}$$

Thus, i can profitably deviate at (p, n). Contradiction.

It remains to show that continuity applies at all t, for all (p, n) on-path such that $p < 1$. Suppose by contradiction that it is not. Let t denote the time at which there is a discontinuity. Because $F_{p,n}$ is increasing and right-differentiable $\lim_{r\to t-} F_{p,n}(r) < F_{p,n}(t)$. By (3), $F_{p(t),n}(0) > 0$. Thus, $F_{p(t),n}$ is discontinuous at 0. Contradiction. □

I now state two technical lemmas (Lemma 3 and Lemma 4), the proofs of which are presented in the Online Appendix. Note that the proof of Lemma 4 relies on Lemma 2.

Lemma 3. *For any (p, n) on-path, $\alpha_n(p) \geq \underline{\alpha}_n(p) \equiv \min\{\beta(1-p)/k_n, 1\}$ and $F'_{p,n}(0+) \leq \overline{f} \equiv \lambda p(\frac{1}{\underline{\alpha}_n(p)} - 1)$.*

Lemma 4. *$\alpha_n(p(s))$ is continuous in s for all (p, n) on path such that $s > 0$.*

Proof of Lemma 2. Assume that $\alpha_n(p) < 1$. By the right continuity and piecewise twice-differentiability of $F_{p,n}$, and by (4), it follows that $\alpha_n(p(s))$ is right-continuous in s. Thus, there exists an $\varepsilon > 0$ and $d > 0$ such that $\alpha_n(p(s)) < 1 - d$ for all $s \in [0, \varepsilon)$. I claim that for all $s \in [0, \varepsilon)$, $V_{p,n} = V_{p,n}(\delta_s)$. Suppose by contradiction that for some $\tilde{s} \in [0, \varepsilon)$,$V_{p,n}(\delta_{\tilde{s}}) < V_{p,n}$. I show that $V_{p,n}(\delta_s)$ is right-continuous in s. By definition,

$$V_{p,n}(\delta_s) = \int_0^s k_n\alpha_n(p(r))d\Psi^i(r) + (N-n)\int_0^s V_{p^i(r),n}d\Psi^{-i}(r) + \\ (1 - \sum_j \Psi^j(s))[k_n\alpha_n(p(s)) - \beta(1-p(s))],$$

where $\Psi^j(s)$ is the first-report distribution that arises when i plays δ_∞ and all $j \neq i$ play $F_{p,n}$. The right-continuity of $V_{p,n}(\delta_s)$ with respect to s then follows from the absolute continuity of Ψ^j (which follows from Lemma 1), and the right-continuity of $\alpha_n(p(s))$ with respect to s, which follows from the right-continuity of $F_{p,n}(s)$.

Given the right continuity of $V_{p,n}(\delta_s)$, there exists some $\varepsilon' \in (0, \varepsilon - \tilde{s})$ and $x > 0$ such that $V_{p,n} - V_{p,n}(\delta_r) > x$ for all $r \in [\tilde{s}, \tilde{s} + \varepsilon']$. I claim there exists some $s^* \in [0, \infty]$ such that $V_{p,n} = V_{p,n}(\delta_{s^*})$. Suppose by contradiction that $V_{p,n} > V_{p,n}(\delta_s)$ for all $s \in [0, \infty]$. It follows from (5) that

$$V_{p,n}(F) = \int_0^\infty V_{p,n}(\delta_s)dF_{p,n}(s) + (1 - \lim_{s\to\infty} F_{p,n})V_{p,n}(\delta_\infty) < V_{p,n},$$

where the strict inequality follows from the assumption that $V_{p,n} > V_{p,n}(\delta_s)$ for all s. Thus, F cannot be an equilibrium strategy. Contradiction.

Now, define the following deviation $\tilde{F}$. This strategy is identical to F, except $\tilde{F}_{p,n}$ shifts all the mass from $[s, s+\varepsilon']$ to s^*. Specifically, when $s^* < \tilde{s}$:

$$\tilde{F}_{p,n}(t) = \begin{cases} F_{p,n}(t) + F_{p,n}(\tilde{s}+\varepsilon) - F_{p,n}(\tilde{s}) & \text{if } t \in [s^*, \tilde{s}] \\ F_{p,n}(\tilde{s}+\varepsilon) & \text{if } t \in (\tilde{s}, \tilde{s}+\varepsilon'] \\ F_{p,n}(t) & \text{otherwise.} \end{cases}$$

Meanwhile, when $s^* > \tilde{s}+\varepsilon'$:

$$\tilde{F}_{p,n}(t) = \begin{cases} F_{p,n}(s) & \text{if } t \in [\tilde{s}, \tilde{s}+\varepsilon] \\ F_{p,n}(t) - [F_{p,n}(\tilde{s}+\varepsilon') - F_{p,n}(\tilde{s})] & \text{if } t \in (\tilde{s}+\varepsilon', s^*) \\ F_{p,n}(t) & \text{otherwise.} \end{cases}$$

Now, by definition:

$$V_{p,n}(\tilde{F}) = V_{p,n}(F) + \int_{\tilde{s}}^{\tilde{s}+\varepsilon'} [V_{p,n}(\delta_{s^*}) - V_{p,n}](\delta_r) dF_{p,n}(r) \geq V_{p,n}(F) + x\varepsilon' > V_{p,n}(F_{p,n}).$$

Thus, $\tilde{F}$ is a profitable deviation. Contradiction.

It remains to show that $V_{p,n} = V_{p,n}(\delta_\infty)$. Suppose by contradiction that $V_{p,n} > V_{p,n}(\delta_\infty)$. It follows that $\lim_{t\to\infty} F_{p,n}(t) = 0$, because otherwise, the firm could profitably deviate by placing no mass on $t = \infty$. Thus, for some $s \in (0, \infty]$, $\lim_{t\to s-} b_n(p(t)) = \infty \Rightarrow \lim_{t\to s-} \alpha_n(p(t)) = 0$, which contradicts Lemma 3. □

Proof of Proposition 1. I begin by showing that $\alpha_n(p) = 1$ whenever $k_n < \beta$ and $p \leq p_n^* \equiv \frac{k_n - \beta}{k_n/n - \beta}$. To this end, fix an n, and suppose that $k_n < \beta$. I first show that for all $q < \frac{\beta - k_n}{\beta}$, $\alpha_n(q) = 1$. For all such q

$$V_{q,n}(\delta_0) = k_n\alpha_n(q) - \beta(1-q) \leq k_n - \beta(1-q) < k_n - \beta(1 - \frac{\beta - k_n}{\beta}) = 0.$$

Since $V_{q,n} \geq V_{q,n}(\delta_\infty) \geq 0$, it follows that $V_{q,n} > V_{q,n}(\delta_0)$. Thus, by Lemma 2, $\alpha_n(q) = 1$. Now, let $q_n^* \equiv \sup\{p | \alpha_n(q) = 1 \text{ for all } q < p\}$. It follows from the above that $q_n^* \geq \frac{\beta - k_n}{\beta}$. I claim that $q_n^* \leq p_n^*$. Suppose by contradiction that $q_n^* < p_n^*$. By Lemma 4, there exists an $\varepsilon > 0$ such that for all $p \in (q_n^*, q_n^* + \varepsilon)$, $\alpha_n(p) < 1$, and thus $V_{p,n} = V_{p,n}(\delta_0) = k_n\alpha_n(p) - \beta(1-p)$. So, $\lim_{p\to q_n^*+} V_{p,n} = k_n - \beta(1-q_n^*)$. By definition of V, because by Lemma 1 $F_{p,n}$ is absolutely continuous, it follows that $V_{p,n}(\delta_\infty)$ is as well, and thus: $\lim_{p\to q_n^*+} V_{p,n}(\delta_\infty) = V_{q_n^*,n}(\delta_\infty) = \frac{k_n q_n^*}{n}$. In order for δ_∞ to not serve as a profitable deviation for $p \in (q_n^*, q_n^* + \varepsilon)$, it must be that for all such p, $V_{p,n}(\delta_0) \geq V_{p,n}(\delta_\infty)$. Taking a limit we obtain that $\lim_{p \to q_n^*+} V_{p,n}(\delta_0) \geq \lim_{p \to q_n^*+} V_{p,n}(\delta_\infty)$. Substituting the limits for $V_{p,n}$ and $V_{p,n}(\delta_\infty)$ above, we obtain that $\frac{k_n q_n^*}{n} \leq k_n - \beta(1 - q_n^*)$. However, $k_n \leq \beta$ and $q_n^* < p$ implies that $\frac{k_n q^*}{n} > k_n - \beta(1 - q^*)$. Contradiction.

Next, we show that $\alpha_n(p) < 1$ whenever $\beta \leq k_n$ or $p > p_n^*$. To this end, assume $\beta \leq k_n$ or $p > p_n^*$. Assume by contradiction that $\alpha_n(p) = 1$. Also assume by induction that if $n < N$, then the statement holds for $n + 1$. First, consider the case where $\alpha_n(q) = 1$ for all $q < p$. By (4), this implies that $F'_{q,n}(0) = 0$ for all $q < p$. Furthermore, by Lemma 1, this implies that $F_{p,n}(s) = 0$ for all $s > 0$, i.e., $F_{p,n} = \delta_\infty$. However, $V_{p,n}(\delta_0) = k_n - \beta(1-p) > \frac{k_n p}{n} = V_{p,n}(\delta_\infty)$, where the strict inequality follows from the assumption that either $\beta \leq k_n$ or $p > p_n^*$. Contradiction.

Next, consider the case where $\alpha_n(q) < 1$ for some $q < p$. By Lemma 4, for all $\varepsilon > 0$ sufficiently small, there exists $\overline{p} < p$ and $\overline{s} > 0$ such that $\alpha_n(\overline{p}) \in (1 - \varepsilon, 1)$ and $\alpha_n(q)$ is strictly increasing on $[\overline{p}(\overline{s}), \overline{p}]$. By Lemma 2, there exists some $\Delta \in (0, s)$ such that

$$V_{\overline{p},n}(\delta_\Delta) = V_{\overline{p},n}(\delta_0). \tag{9}$$

By definition,

$$\begin{aligned}
&V_{\overline{p},n}(\delta_\Delta) = \int_0^\Delta k_n \alpha_n(\overline{p}(s)) d\Psi^i(s) + (N-n) \int_0^\Delta V_{\overline{p}^i(s),n+1} d\Psi^{-i}(s) + \\
&(1 - \sum_j \Psi^j(\Delta))[k_n \alpha_n(\overline{p}(\Delta)) - \beta(1 - \overline{p}(\Delta))],
\end{aligned}$$

where Ψ is the first-report distribution associated with the strategy profile where i plays δ_Δ and all $j \neq i$ play $F_{p,n}$. Meanwhile,

$$\begin{aligned}
&V_{\overline{p},n}(\delta_0) = \int_0^\Delta k_n \alpha_n(\overline{p}) d\Psi^i(s) + (N-n) \int_0^\Delta k_n \alpha_n(\overline{p}) - \beta(1 - \overline{p}^i(s)) d\Psi^{-i}(s) \\
&+ (1 - \sum_j \Psi^j(\Delta))(k_n \alpha_n(\overline{p}) - \beta(1 - \overline{p}(\Delta)).
\end{aligned}$$

Thus, in order for (9) to hold, for some $r \in (0, \overline{s})$,

$$k_n \alpha_n(\overline{p}) - \beta(1 - \overline{p}^i(r)) < V_{\overline{p}^i(r),n+1}. \tag{10}$$

First, consider the case where $\alpha_{n+1}(\overline{p}^i(r))) < 1$. Then, for $\varepsilon > 0$ sufficiently small

$$V_{\overline{p}^i(r),n+1} = V_{\overline{p}^i(r),n+1}(\delta_0) = k_{n+1}\alpha_{n+1}(\overline{p}^i(r)) - \beta(1 - \overline{p}^i(r)) < k_n \alpha_n(\overline{p}) - \beta(1 - \overline{p}^i(r)),$$

Thus, equation (10) is violated. Contradiction. Next, consider the case where $\alpha_{n+1}(\overline{p}^i(r)) = 1$ and $\beta < k_n$. By the inductive assumption, it follows that $\alpha_{n+1}(q) = 1$ for all $q \leq \overline{p}^i(s)$. Thus, $F_{\overline{p}^i(s),n+1} = \delta_\infty$. So, we have that for ε sufficiently small:

$$V_{\overline{p}^i(r),n+1} = \frac{k_{n+1}\overline{p}^i(r)}{N-n} \leq \overline{p}^i(r)k_n\alpha_n(\overline{p}) + (1-\overline{p}^i(r))k_n\alpha_n(\overline{p}) - \beta) = k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}^i(r)).$$

Again, this is a contradiction of (10).

Finally, consider the case where $\alpha_{n+1}(\overline{p}^i(r)) = 1$ and $\beta \geq k_n$. Recall that $\alpha_n(q) = 1$ for all $q \geq p_n^*$. Thus, because $\alpha_n(\overline{p}) < 1$, it follows from (4) that $\alpha_n(\overline{p}(s))$ must be strictly increasing in s for some $s > r$. Formally, let $r' \equiv \inf\{s > r | \alpha_n(\overline{p}(s))$ is strictly increasing$\}$. First, I claim that

$$k_n\alpha_n(\overline{p}(r')) - \beta(1-\overline{p}^i(r')) < V_{\overline{p}^i(r'),n+1}. \tag{11}$$

By the inductive assumption, since $\alpha_{n+1}(\overline{p}^i(r)) = 1$, it must be that $\alpha_{n+1}(q) = 1$ for all $q < \overline{p}^i(r)$. Because $\alpha_n(\overline{p}(s))$ is weakly decreasing in s for $s \in [r, r']$, it follows by definition of $\overline{p}^i(s)$ that $\overline{p}^i(s) < \overline{p}^i(r)$ for all $s \in [r, r']$. Thus, for all $s \in [r, r']$ $V_{\overline{p}^i(s),n+1} = \frac{k_{n+1}\overline{p}^i(s)}{N-n}$. Then, for all $s \geq r$,

$$k_n\alpha_n(\overline{p}(s)) - \beta(1-\overline{p}^i(s)) < V_{\overline{p}^i(s),n+1} \Leftrightarrow \overline{p}^i(s) < \frac{\beta - k_n\alpha_n(\overline{p}(s))}{\beta - k_{n+1}/(N-n)}.$$

Now, because $\alpha_n(\overline{p}(s))$ is strictly decreasing on $s \in [0, r]$,

$$k_n\alpha_n(\overline{p}(r)) - \beta(1-\overline{p}^i(r)) < k_n\alpha_n(\overline{p}) - \beta(1-\overline{p}^i(r)) < V_{\overline{p}^i(r),n+1}.$$

where the second inequality holds for the same reason as (10). Thus we have

$$\overline{p}^i(r') < \overline{p}^i(r) < \frac{\beta - k_n\alpha_{n+1}(\overline{p}(r))}{\beta - k_{n+1}/(N-n)} < \frac{\beta - k_n\alpha_{n+1}(\overline{p}(r'))}{\beta - k_{n+1}/(N-n)},$$

which implies (11).

It follows from this that there exists an $r'' > r'$ such that for all $s \in [r', r'']$, $\alpha_n(\overline{p}(s))$ is weakly decreasing and $V_{\overline{p}^i(s),n+1} > k_n\alpha_n(\overline{p}(r')) - \beta(1-p^i(s))$. I now claim that

$V_{\overline{p}(r'),n}(\delta_0) < V_{\overline{p}(r'),n}(\delta_{r''-r'})$. To see why, note that by definition,

$$V_{\overline{p}(r'),n}(\delta_{r''-r'}) - V_{\overline{p}(r'),n}(\delta_0) = \int_{r'}^{r''} k_n[\alpha_n(p(s)) - \alpha_n(p(r'))]d\Psi^i(s) + \int_{r'}^{r''} [V_{p^i(s),n+1} - (k_n\alpha_n(p(r')) - \beta(1-p^i(s)))]d\Psi^{-i}(s) + \sum_j (\Psi^j(r'') - \Psi^j(r'))k_n(\alpha_n(p(r'')) - k_n\alpha_n(p(r'))).$$

Since $\alpha_n(p(s)) \geq \alpha_n(p(r'))$ and $V_{p^i(s),n+1} > k_n\alpha_n(p(r')) - \beta(1-p^i(s))$ for all $s \in [r', r'']$, it follows that $V_{\overline{p}(r'),n}(\delta_{r''-r'}) - V_{\overline{p}(r'),n}(\delta_0) > 0$. This contradicts Lemma 2. □

Proof of Proposition 2. Proof by induction. Fix an n, and assume that $\alpha_m(p)$ satisfies the above for all $m > n$ such that (p, m) is on-path. I begin by showing that (ODE) must hold whenever $\alpha_n(p) < 1$. To this end, assume that $\alpha_n(p) < 1$. By Lemma 2, there exists an $\varepsilon > 0$ such that for all $\Delta \in (0, \varepsilon)$,

$$\frac{V_{p,n}(\delta_\Delta) - V_{p,n}(\delta_0)}{\Delta} = 0. \tag{12}$$

By definition, $V_{p,n}(\delta_0) = k_n\alpha_n(p) - \beta(1-p)$. Meanwhile,

$$V_{p,n}(\delta_\Delta) = \int_0^\Delta k_n\alpha_n(p(s))d\Psi^i(s) + (N-n)\int_0^\Delta V_{p^{-i}(s),n+1}d\Psi^{-i}(s) + (1 - \sum_j \lim_{s\to\Delta-} \Psi^j(s))[k_n\alpha_n(p(\Delta)) - \beta(1-p(\Delta))],$$

where Ψ is the first-report distribution associated with the strategy profile in which i plays δ_∞ and all $j \neq i$ play the equilibrium strategy $F_{p,n}$. Specifically, for all $s > 0$,

$$\Psi^i(s) = p\lambda \int_0^s e^{-\lambda r(N-n+1)}(1 - F_{p,n}(r))^{N-n}dr$$

$$\Psi^{-i}(s) = p\int_0^s e^{-\lambda r(N-n)}(1 - F_{p,n}(r))^{N-n-1}d(-e^{-\lambda r}(1 - F_{p,n}(r))) + (1-p)\int_0^s (1 - F_{p,n}(r))^{N-n-1}dF_{p,n}(r).$$

It follows from Lemma 1 that, for all j, Ψ^j is absolutely continuous on $[0, \Delta)$, i.e., $\Psi^j(s) = \int_0^s \psi^j(r)dr$. where ψ^i and ψ^{-i} are given by the following:

$$\psi^i(r) = p\lambda e^{-\lambda r(N-n+1)}(1 - F_{p,n}(r))^{N-n}$$

$\psi^{-i}(s) = pe^{-\lambda s(N-n+1)}(\lambda + F'_{p,n}(s+) - \lambda F_{p,n}(s))(1-F_{p,n}(s))^{N-n-1} + (1-p)(1-F_{p,n}(s))^{N-n-1}F'_{p,n}(s+)$.

Substituting the expressions for both $V_{p,n}(\delta_0)$ and $V_{p,n}(\delta_\Delta)$ into (12) and rearranging, we obtain that for all $\Delta \in (0, \varepsilon)$,

$$K_1(\Delta) + K_2(\Delta) + K_3(\Delta) = 0 \tag{13}$$

where

$$K_1(\Delta) \equiv \frac{\int_0^\Delta k_n[(\alpha_n(p(s)) - \alpha_n(p)) + \beta(1-p)]\psi^i(s)ds}{\Delta}$$

$$K_2(\Delta) \equiv \frac{(N-n)\int_0^\Delta [V_{p^{-i}(s),n+1} - k_n\alpha_n(p) + \beta(1-p)]\psi^{-i}(s)ds}{\Delta}$$

$$K_3(\Delta) \equiv \frac{(1 - \sum_j \lim_{s\to\Delta-} \Psi^j(\Delta))[k_n(\alpha_n(p(\Delta)) - \alpha_n(p)) + \beta(p(\Delta) - p)]}{\Delta}.$$

Now, we consider $\lim_{\Delta\to 0+}$ of $K_1(\Delta)$, $K_2(\Delta)$, and $K_3(\Delta)$ separately. For $K_1(\Delta)$, it follows from L'Hôpital's Rule, together with the continuity of $\alpha_n(p(\Delta))$ (Lemma 4) and $\psi^i(\Delta)$ in Δ that

$$\lim_{\Delta\to 0+} K_1(\Delta) = \beta(1-p)\psi^i(0) = \beta(1-p)p\lambda.$$

For $K_2(\Delta)$, it again follows from L'Hôpital's Rule, together with the right-continuity of $V_{p^{-i}(\Delta),n+1}$ in Δ that

$$\lim_{\Delta\to 0+} K_2(\Delta) = (N-n)[V_{p^{-i},n+1} - k_n\alpha_n(p) + \beta(1-p)](\frac{\lambda p}{\alpha_n(p)}).$$

For $K_3(\Delta)$, by the continuous differentiability of $\Psi^j(s)$ that $\lim_{\Delta\to 0+}\sum_j \lim_{s\to\Delta-}\Psi^j(s) = 0$. Thus, it follows from the right-differentiability of $\alpha_n(p(\Delta))$ in Δ that

$$\lim_{\Delta\to 0+} K_3(\Delta) = p'(\Delta)\Big|_{\Delta=0+}[k_n\alpha'_n(p) + \beta] = -\lambda p(N-n+1)(1-p)[k_n\alpha'_n(p) + \beta].$$

Since we have shown that $\lim_{\Delta\to 0+} K_1(\Delta)$, $\lim_{\Delta\to 0+} K_2(\Delta)$, and $\lim_{\Delta\to 0+} K_3(\Delta)$ exist, and are given by the above expressions, it follows from (13) that

$$\lim_{\Delta\to 0+} K_1(\Delta) + \lim_{\Delta\to 0+} K_2(\Delta) + \lim_{\Delta\to 0+} K_3(\Delta) = 0.$$

Substituting the above expressions for $K_1(\Delta)$, $K_2(\Delta)$ and $K_3(\Delta)$, we obtain (ODE).

Now, we wish to establish that (ODE) must hold whenever $k_n \geq \beta$ or $p > p_n^*$. It follows from Proposition 1 that $\alpha_n(p) < 1$, and thus by the above, (ODE) must hold.

Finally, we establish the two limit conditions presented in the proposition. We begin by establishing that when $k_n \geq \beta$, $\lim_{p\to 0+} \alpha_n(p) = \beta/k_n$. To this end, first note by Lemma 2 that for all $p > 0$, $V_{p,n}(\delta_0) = V_{p,n}(\delta_\infty)$. Note further that $\lim_{p\to 0+} V_{p,n}(\delta_\infty) = 0$. Thus, $\lim_{p\to 0+} V_{p,n}(\delta_0) = \lim_{p\to 0+} k_n\alpha_n(p) - \beta = 0$, and therefore, $\lim_{p\to 0+} \alpha_n(p) = \frac{\beta}{k_n}$. Next, let us consider the case where $k_n < \beta$. That $\lim_{p\to p_n^*+} \alpha_n(p) = 1$ follows from Lemma 4, since by Proposition 1, $\alpha_n(p_n^*) = 1$. □

I now I define a problem (P) on α. I first show that α constitutes an equilibrium if and only if it satisfies (P) and (SC) is optimal for a firm who has privately confirmed the state (Lemma 5). I then show that assuming α satisfies (P), it is indeed optimal for a firm who has confirmed the state to abide by (SC) (Lemma 6). The proofs of Lemma 5 and Lemma 6 are relegated to the Online Appendix. Thus, existence and uniqueness of an equilibrium (Theorem 1) will reduce to establishing a unique solution to (P).

Definition 3. α is a solution to (P) if it satisfies the following for all n and $p \in (0, 1]$:

1. If $k_n < \beta$ and $p \leq p_n^* \equiv \frac{k_n-\beta}{k_n/n-\beta}$, then $\alpha_n(p) = 1$.
2. If $k_n \geq \beta$ or $p < p_n^*$, then α satisfies (ODE), with limit condition $\lim_{p\to 0+} \alpha_n(p) = \beta/k_n$ if $k_n \geq \beta$ and $\lim_{p\to p_n *+} \alpha_n(p) = 1$ if $k_n < \beta$.
3. $\alpha_n(1) = 0$.

Lemma 5. *(α, F) is an equilibrium if and only if at all (p, n) on-path, α is both consistent with F and a solution to (P).*

Lemma 6. *Suppose that α is a solution to (P). Then, $F_{1,n}(0) = 1$ is optimal for all n.*

Proof of Theorem 1. Fix an n. Assume by induction that there exists a unique solution to (P) for all $m > n$. We wish to show that there exists a unique solution to (P) for n. It suffices to show there exists a unique solution to the following two problems, when $\beta \leq k_n$ and $\beta > k_n$, respectively:

$$\text{(ODE-}i\text{) is satisfied on } [0, 1)\text{, and } \alpha_n(0) = \beta/k_n \qquad \text{(BVP: } \beta \leq k_n\text{)}$$

$$\text{(ODE-}i\text{) is satisfied on } (0, p_n^*]\text{, and } \alpha_n(p^*) = 1. \qquad \text{(BVP: } \beta \geq k_n\text{)}$$

where

$$\alpha_n'(p) = -\frac{1}{k_n(1-p)\alpha_n(p)} \frac{N-n}{N-n+1}[k_n\alpha_n(p) - \tilde{V}_{p^i,n+1} - \beta(1-\alpha_n(p))(1-p)], \quad \text{(ODE')}$$

and

$$\tilde{V}_{p^i,n+1} = \begin{cases} V_{p^i,n+1} & \text{if } p^i \in (0,1) \\ 0 & \text{if } p^i \geq 1. \end{cases}$$

We do this by invoking the Picard existence and uniqueness theorem, and thus begin by establishing that the right-hand side of (ODE-i) is Lipschitz continuous in $\alpha_n(p)$ and continuous in p for $p \in [-\varepsilon, 1)$ and $\alpha_n(p) \in [c, 1+\varepsilon]$ for any $c > 0$ and some $\varepsilon > 0$. Since $p^i \equiv \alpha_n(p) + (1 - \alpha_n(p))p$, it suffices to show that $\tilde{V}_{p^i,n+1}$ is Lipschitz continuous in p^i for $p^i \geq 0$. In the case where $n = N$, $\tilde{V}_{p^i,n+1} = 0$ for all p^i, and this is immediate. Next, suppose $n > 1$. First, consider the case where $k_{n+1} \geq \beta$. It follows from Lemma 2 that:

$$\tilde{V}_{p^i,n+1} = \begin{cases} k_n \alpha_{n+1}(p^i) - \beta(1-p^i) & \text{if } p^i < 1 \\ 0 & \text{if } p^i \geq 1. \end{cases}$$

Because $\tilde{V}_{p^i,n+1}$ is continuously differentiable in p^i when $p^i \neq 1$, to establish that it is Lipschitz continuous it suffices to show that $\lim_{p^i \to 1-} V_{p^i,n+1} = 0$. Suppose this does not hold, by contradiction. Because $\alpha_{n+1}(\cdot)$ satisfies (ODE), this implies that $\lim_{p^i \to 1-} \alpha'_{n+1}(p^i) = \infty$. This implies that $\lim_{p^i \to 1} \alpha_{n+1}(p^i) = \infty$, and thus that (ODE) is not satisfied at $p^i = 1$. Contradiction. Next, consider the case where $k_{n+1} < \beta$:

$$\tilde{V}_{p^i,n+1} = \begin{cases} k_{n+1} p^i/(N-n) & \text{if } p^i < p^*_{n+1} \\ k_n \alpha_{n+1}(p^i) - \beta(1-p^i) & \text{if } p^i \in (p^*_{n+1}, 1) \\ 0 & \text{if } p^i = 1. \end{cases}$$

By the same reasoning as above, $\tilde{V}_{p^i,n+1}$ is Lipschitz continuous for all $p^i > p^*_{n+1}$. Furthermore, Lipschitz continuity holds for $p^i < p^*_{n+1}$. To show that Lipschitz continuity holds for all p^i, it suffices to show $\tilde{V}_{\cdot,n+1}$ is differentiable at p^*_{n+1}. To this end, we take the left- and right- derivative of $\tilde{V}_{\cdot,n+1}$ at p^*_{n+1} and show they are equal:

$$\frac{d}{dp}\tilde{V}_{p^*_{n+1}-,n+1} = \frac{k_{n+1}}{N-n} = \frac{d}{dp}\tilde{V}_{p^*_{n+1}+,n+1}$$

Now, we show that there exists a unique solution for both (BVP: $\beta \leq k_n$) and (BVP: $\beta \geq k_n$) in some neighborhood of their respective boundary conditions. By the Picard Theorem, this follows immediately from our above-established result that the right-hand side of (ODE-i) is Lipschitz continuous in $\alpha_n(p)$ and continuous in p in some neighborhood of the boundary conditions $(\alpha_n(p) = 1, p = p_n^*)$ and $(\alpha_n(p) = \beta/k_n, p = 0)$.

Next, we seek to establish global existence and uniqueness of solutions to both (BVP: $\beta \leq k_n$) and (BVP: $\beta \geq k_n$). First, consider (BVP: $\beta \geq k_n$). The argument for (BVP: $\beta \leq k_n$) follows analogously. Let $[p^*, \overline{p})$ denote the largest right-open interval such that existence and uniqueness are both satisfied. Assume by contradiction that $\overline{p} < 1$. Let $\alpha_n(p)$ denote the solution along this interval.

We begin by showing that on this interval, $\alpha_n(p) \in (\underline{\alpha}, 1]$, where $\underline{\alpha} > 0$ is some constant. The upper bound is established as follows: suppose by contradiction that $\alpha_n(p) > 1$ somewhere on the interval. By the continuous differentiability of α_n along the interval, there must exist some $q < p$ such that $\alpha_n(q) = 1$ and $\alpha_n'(q) \geq 0$. However, it follows from (ODE-i) that $\alpha_n'(q) = -\frac{1}{k_n(1-q)} \frac{N-n}{N-n+1}[k_n - \tilde{V}_{p^i,n+1}] < 0$, where the strict inequality follows from the fact that $\tilde{V}_{p^i,n+1} \leq k_{n+1} < k_n$. Contradiction. The lower bound is established as follows: suppose by contradiction that such a lower bound does not exist. Then, again by the continuous differentiability of α_n along the interval, there exists some $\hat{p} \in [p_n^*, \overline{p})$ such that $\lim_{p \to \hat{p}-} \alpha_n(p) = 0$ and $\alpha_n(p) > 0$ for all $p < \hat{p}$. However, it then follows from (ODE) that $\lim_{p \to \hat{p}-} \alpha_n'(p) = \infty$. Thus, (ODE-$i$) is not satisfied on $[p_n^*, \overline{p})$. Contradiction.

Having established that on $[p^*, \overline{p})$, $1 \geq \alpha_n(p) > \underline{\alpha} > 0$, it follows from (ODE-$i$), and the observation that $\tilde{V}_{p^i,n+1}$ is bounded, that α_n' is also bounded on this range. Thus, it follows that $\lim_{p \to \overline{p}-} \alpha_n(p) \equiv \overline{\alpha} > 0$ exists.

Now, consider the following modified boundary value problem, which is identical to (BVP: $\beta \geq k_n$), except with boundary condition $(\overline{p}, \overline{\alpha})$. Recall we have shown that (ODE-i) is Lipschitz continuous in $\alpha_n(p)$ and continuous in p in some neighborhood of $(\overline{p}, \overline{\alpha})$. Thus, we can again apply the Picard Theorem to obtain that there exists a unique solution to the modified boundary value problem in some neighborhood of $(\overline{p}, \overline{\alpha})$. I.e., there exists some $\varepsilon > 0$ such that there is a unique solution $\tilde{\alpha}_n(p)$ on interval $(\overline{p} - \varepsilon, \overline{p} + \varepsilon)$. We now "paste" this solution $\tilde{\alpha}_n$ with α_n. Let

$$\hat{\alpha}_n(p) = \begin{cases} \alpha_n(p) & \text{if } p \in [p_n^*, \overline{p}) \\ \tilde{\alpha}_n(p) & \text{if } p \in [\overline{p}, \overline{p} + \varepsilon). \end{cases}$$

$\hat{\alpha}_n(p)$ is a unique solution to (BVP: $\beta \geq k_n$) on $[p_n^*, \overline{p} + \varepsilon)$, which contradicts our earlier assumption that $[p^*, \overline{p})$ was the largest right-open interval such that existence and uniqueness are satisfied. Contradiction. □

Proof of Proposition 3 and Proposition 4. Let us begin by showing that $\alpha_n(p)$ is weakly decreasing in p for all (p, n) on-path. By Lemma 5, it follows that when $k_N < \beta$, $\alpha_N(p) = 1$ for all p, and when $k_N \geq \beta$, $\alpha'_N(p) = 0$ for all p. Thus, $\alpha_N(p)$ is constant in p. Now, consider the case where $n < N$. Assume by induction that $\alpha_{n+1}(p)$ is weakly decreasing in p whenever $(p, n+1)$ is on path.

Assume by contradiction that there exists some $\overline{p}$ such that α_n is strictly increasing. By Lemma 5, $\alpha'_n(p) = 0$ whenever $\beta \geq k_n$ and $p \geq p^*_n$. Thus it must be that $\beta < k_n$ or $\overline{p} > p^*_n$. In this case, (ODE) must be satisfied. Now define the function $X(p)$ as follows:

$$X(p) \equiv k_n \alpha_n(p) - \beta(1 - p^i) - V_{p^i,n+1}. \tag{14}$$

Whenever (ODE) is satisfied, the following holds:

$$\alpha'_n(p) > (=) 0 \text{ if and only if } X(p) < (=)\ 0. \tag{15}$$

Thus, $X(\overline{p}) < 0$. Now, I claim that there exists $\underline{p} < \overline{p}$ such that $\lim_{p \to \underline{p}+} X(p) \geq 0$. To establish this, first suppose $k_n \geq \beta$. In this case,

$$\lim_{p \to 0+} X(p) = (k_n + \beta) \lim_{p \to 0+} \alpha_n(p) - \beta - \lim_{p \to 0+} V_{\alpha_n(p),n+1}. \tag{16}$$

When $\lim_{p \to 0+} \alpha_{n+1}(\alpha_n(p)) < 1$, it follows from Lemma 2 that

$$\lim_{p \to 0+} V_{\alpha_n(p),n+1} = k_{n+1} \alpha_{n+1}(\beta/k_n) - \beta(1 - \beta/k_n).$$

Substituting this into (16), we obtain $\lim_{p \to 0+} X(p) = \beta - k_{n+1}\alpha_{n+1}(\beta/k_n)$. In the case where $k_{n+1} < \beta$, it follows directly that $\lim_{p \to 0+} X(p) \geq 0$. Otherwise, if $k_{n+1} \geq \beta$, then because $\lim_{p \to 0+} \alpha_{n+1}(p) = \beta/k_{n+1}$, it follows from the inductive assumption that $\alpha_{n+1}(p) \leq \beta/k_{n+1}$ for all p, and thus that $\lim_{p \to 0+} X(p) \geq 0$.

When $\lim_{p \to 0+} \alpha_{n+1}(\alpha_n(p)) = 1$, it follows from the inductive assumption that $\alpha_{n+1}(q) = 1$ for all $q \geq \lim_{p \to 0+} \alpha_n(p)$. Thus,

$$\lim_{p \to 0+} V_{p^i,n+1} = \lim_{p \to 0+} V_{p^i,n+1}(\delta_\infty) = \frac{k_{n+1}}{N-n} \frac{\beta}{k_n}.$$

Substituting into the above expression for $\lim_{p \to 0+} V_{p^i,n+1}$, we obtain $\lim_{p \to 0+} X(p) \geq 0$, implying by Lemma 5 that $k_{n+1} \geq \beta$.

Next, consider the case where $k_n < \beta$. In this case, $\lim_{p \to p^*_n+} X(p) =$

$k_n - \lim_{p^i \to 1-} V_{p^i,n+1}$. If $\lim_{p^i \to 1-} \alpha_{n+1}(p^i) < 1$, then by Lemma 2,

$$\lim_{p^i \to 1-} V_{p^i,n+1} = \lim_{p^i \to 1-} V_{p^i,n+1}(\delta_0) = k_{n+1} \lim_{p^i \to 1-} \alpha_{n+1}(p^i) < k_n.$$

Thus, we obtain that $\lim_{p \to p_n^*+} X(p) > 0$. Meanwhile, if $\lim_{p^i \to 1-} \alpha_{n+1}(p^i) = 1$, by the inductive assumption, $\alpha_{n+1}(p) = 1$ for all p. Thus,

$$\lim_{p^i \to 1-} V_{p^i,n+1} = \lim_{p^i \to 1-} V_{p^i,n+1}(\delta_\infty) = \frac{k_{n+1}}{N-n}.$$

So in this case as well, $\lim_{p \to p_n^*+} X(p) > 0$. We have thus shown that there always exists $\underline{p} < \overline{p}$ such that $\lim_{p \to \underline{p}+} X(p) \geq 0$.

Because $X(\overline{p}) < 0$, there must exist some $q \in [\underline{p}, \overline{p}]$ such that $X(q) < 0$ and $X'(q) < 0$. Differentiating X, we obtain

$$X'(q) = k_n \alpha_n'(q) + \beta((1-q)\alpha_n'(q) + (1 - \alpha_n(q))) - \frac{d}{dq} V_{q^i,n+1}. \tag{17}$$

First, consider the case where $\alpha_{n+1}(q^i) < 1$. By Lemma 2,

$$V_{q^i,n+1} = V_{q^i,n+1}(\delta_0) = k_{n+1}\alpha_{n+1}(q^i) - \beta(1 - q^i). \tag{18}$$

Substituting this into (17), we obtain $X'(q) = k_n\alpha_n'(q) - k_{n+1}\alpha_{n+1}'(q^i)((1-q)\alpha_n'(q) + (1 - \alpha_n(q)))$. Because $X(q) < 0$, it follows from (15) that $\alpha_n'(q) > 0$. Furthermore, by the inductive assumption, $\alpha_{n+1}'(q^i) \leq 0$. Thus, $X'(q) > 0$. Contradiction.

Next, consider the case where $\alpha_{n+1}(q^i) = 1$. By the inductive assumption, $\alpha_{n+1}(p) = 1$ for all $p \leq q^i$. Thus, $V_{q^i,n+1} = V_{q^i,n+1}(\delta_\infty) = \frac{k_{n+1}q^i}{N-n}$. Substituting into (17):

$$X'(q) = k_n\alpha_n'(q) + (\beta - \frac{k_{n+1}}{N-n})((1-q)\alpha_n'(q) + (1 - \alpha_n(q))). \tag{19}$$

Because $\alpha_{n+1}(q^i) = 1$, by Lemma 5, $\beta \geq k_{n+1}$. Thus, $X'(q) > 0$. Contradiction.

Next, we show that if $k_N \geq \beta$, then $\alpha_n(p) = \beta/k_n$. Assume that $k_N \geq \beta$. First consider $n = N$. By Lemma 5, $\alpha_n'(p) = 0$ for all p on-path, and thus, $\alpha_N(p)$ is constant in p. Since Lemma 5 also states that $\lim_{p \to 0+} k_N \alpha_N(p) = \beta$, it must be that $\alpha_N(p) = \beta/k_N$ for all p. Now, consider $n < N$. Assume by induction that $\alpha_{n+1}(p) = \beta/k_{n+1}$ for all p. We begin by showing that $\alpha_n(p)$ is constant in p. Since $k_n \geq \beta$, by Lemma 5, (ODE) must hold at all p. By (15), showing $\alpha_n(p)$ is constant in p is equivalent to showing that $X(p) = 0$. To establish this, I begin by claiming that $V_{p^i,n+1} = V_{p^i,n+1}(\delta_0)$.

In the case where $k_{n+1} > \beta$, it follows from Proposition 1 that $\alpha_{n+1}(p^i) < 1$, and thus this follows from Lemma 2. In the case where $k_{n+1} = \beta$, $k_m = \beta$ for all $m \geq n+1$, and by Proposition 1, $\alpha_m(p) = 1$ for all p. Thus, $V_{p,n+1}(\delta_s) = p\beta$. for all $\delta \in [0, \infty]$ and all p. Thus, $V_{p^i,n+1} = V_{p^i,n+1}(\delta_0)$. Having established that $V_{p^i,n+1} = V_{p^i,n+1}(\delta_0)$, we have: $V_{p^i,n+1} = k_{n+1}\alpha_{n+1}(p^i) - \beta(1-p^i) = \beta p^i$. Substituting this into (14), we obtain $X(p) = k_n\alpha_n(p) - \beta$. Since $\alpha_n(p)$ is weakly decreasing, $\alpha_n(p) \leq k_n/\beta$ for all p, and thus $X(p) \leq 0$. Separately, by (15) $\alpha_n(p)$ weakly decreasing implies that $X(p) \geq 0$. Combining these inequalities, we have $X(p) = 0$.

Finally, I show that $k_N < \beta$ implies that $\alpha_n'(p) < 0$ whenever $\alpha_n(p) < 1$. Suppose $k_N < \beta$, and suppose by contradiction that at some q such that $\alpha_n(q) < 1$, $\alpha_n'(q) = 0$. It follows from (15) that $X(q) = 0$. First, suppose $\alpha_{n+1}(q^i) = 1$. Recall from (19) that $X'(q) = (\beta - \frac{k_{n+1}}{N-n})(1-\alpha_n(q))$. Now, I claim that $\beta > \frac{k_{n+1}}{N-n}$. When $n = N-1$, this follows from the assumption that $k_N < \beta$. When $n < N-1$, because $\alpha_{n+1}(q^i) = 1$, this is a result of Proposition 1. Thus, $X'(q) > 0$. Since $X(q) = 0$ for some $p < q$, $X(p) < 0$. By (17), $\alpha_n'(p) > 0$. This contradicts $\alpha_n(p)$ being weakly decreasing in p. Next, suppose $\alpha_{n+1}(q^i) < 1$. By (18), $X'(q) = -k_{n+1}\alpha_{n+1}'(q)[1-\alpha_n(q)] > 0$. This implies that there exists some $p < q$ such that $X(p) < 0$ and thus that $\alpha'(p) > 0$. Contradiction. □

Proof of Corollary 3. It suffices to show that $\lim_{p\to 0+} b_{n+1}(\tilde{p}) - b_n(p) > 0$. It follows from Proposition 2 and (4) that $\lim_{p\to 0+} b_n(p) = 0$. Also, $\lim_{p\to 0+} \tilde{p} = \lim_{p\to 0+} \alpha_n(p) = \beta/k_n$, where the final equality follows from Proposition 2. Thus, $\lim_{p\to 0+} b_{n+1}(\tilde{p}) = b_{n+1}(\lim_{p\to 0+} \tilde{p}) = b_{n+1}(\beta/k_n)$, Thus, $\lim_{p\to 0+}[b_{n+1}(\tilde{p}) - b_n(p)] = b_{n+1}(\beta/k_n) > 0$. □

Commitment solution

Here, we seek the optimal solution to the monopoly case of the baseline model in which the firm has the ability to commit to a reporting strategy. Rather than F and α being determined simultaneously as they are in equilibrium, the firm chooses its strategy F before α is determined. Thus, in the commitment case, the credibility function is a function of the firm's strategy. We formalize this dependence by denoting the firm's credibility function as α_F. For convenience, I will be writing all functions as a function of calendar time t, rather than the common belief p.

The firm's objective is to choose a permissible strategy $F \in \mathcal{F}$ which maximizes its expected payoff over the course of the game. Specifically, its problem is given by:

$$\max_{F\in\mathcal{F}} \int_0^\infty [\alpha_F(t) - \beta(1-p(t))(1-\alpha_F(t))]d\Psi(t), \tag{20}$$

where, as in the baseline setup, $\Psi(t)$ denotes probability that the firm reports before time t under strategy F. It is useful for us to cast this problem as a choice of an optimal credibility function α, rather than an optimal strategy F. To this end, I begin with a useful observation, which is analogous to Lemma 1, except under the commitment:

Lemma 7. *F must be continuous in equilibrium.*

We omit a proof for this claim, as it follows analogously to the proof for Lemma 1. It follows immediately from Lemma 7 that in equilibrium, both the firm's strategy F and the corresponding commitment function, α_F, are defined by the right-hazard rate $b(t)$ of the firm's strategy. That is, $\alpha_F(t) = \frac{\lambda p(t)}{\lambda p(t)+b(t)}$. It further follows that Ψ is continuous and can thus be written as a function of α_F as follows:

$$\Psi(t) = 1 - e^{-\int_0^t (b(s)+p(s)\lambda)ds} = 1 - e^{-\int_0^t \frac{\lambda p(s)}{\alpha_F(s)}ds}.$$

We can now cast the optimization problem given by (20) as one over α_F:

$$\max_{\alpha_F} \int_0^\infty \lambda p(t)[1-\beta(1-p(t))(\frac{1}{\alpha_F(t)}-1)]e^{-\int_0^t \frac{\lambda p(s)}{\alpha_F(s)}ds}.$$

I now prove Claim 2. Formally, I show that $\alpha_F(t) = 1$ for all t. In the proof that follows, I let $V(t, \alpha_F)$ denote the firm's value at time t given that it has chosen α_F.

Proof of Claim 2. Assume not, by contradiction. Then there exists a t^* such that $\alpha_F(t^*) < 1$. It follows from Lemma 7, and the assumption that F is right-continuously differentiable, that α_F must be right-continuous. Thus, there must exist a $\hat{\alpha} < 1$ and $\varepsilon > 0$ such that $\alpha_F(t) < \hat{\alpha}$ for all $t \in [t^*, t^* + \varepsilon]$. For any α_F, we can write:

$$V(0,\alpha_F) = \int_0^{t^*+\varepsilon} \lambda p(t)[1-\beta(1-p(t))](\frac{1}{\alpha_F(t)}-1)e^{-\int_0^t \frac{\lambda p(s)}{\alpha_F(s)}ds}dt + e^{-\int_0^{t^*+\varepsilon} \frac{\lambda p(s)}{\alpha_F(s)}ds} V(t^*+\varepsilon, \alpha_F).$$

Now, consider the deviation $\tilde{\alpha}_F$:

$$\tilde{\alpha}_F(t) = \begin{cases} 1 & \text{if } t \in [t^*, t^*+\varepsilon] \\ \alpha_F(t) & \text{otherwise.} \end{cases}$$

Thus,

$$\begin{aligned} V(0,\alpha_F) = V(0,\tilde{\alpha_F}) &+ \int_{t^*}^{t^*+\varepsilon} \lambda p(t)[1-\beta(1-p(t))(\frac{1}{\alpha_F(t)}-1)]e^{-\int_0^t \frac{\lambda p(s)}{\alpha_F(s)}ds}dt \\ -\int_{t^*}^{t^*+\varepsilon} \lambda p(t) e^{-\int_0^t \lambda p(s)ds}dt &+ (e^{-\int_{t^*}^{t^*+\varepsilon} \frac{\lambda p(s)}{\alpha_F(s)}ds} - e^{-\int_{t^*}^{t^*+\varepsilon} \lambda p(s)ds})V(t^*+\varepsilon, \alpha_F) \end{aligned} \quad (21)$$

We note the following two inequalities:

$$\int_{t^*}^{t^*+\varepsilon} \lambda p(t)[1-\beta(1-p(t))(\frac{1}{\alpha_F(t)}-1)]e^{-\int_0^t \frac{\lambda p(s)}{\alpha_F(s)}ds}dt < \int_{t^*}^{t^*+\varepsilon} \lambda p(t)e^{-\int_0^t \lambda p(s)ds}dt$$

$$e^{-\int_{t^*}^{t^*+\varepsilon} \frac{\lambda p(s)}{\alpha_F(s)}ds} - e^{-\int_{t^*}^{t^*+\varepsilon} \lambda p(s)ds} < 0$$

These two inequalities combined with (21) yields $V(0,\alpha_F) < V(0,\tilde{\alpha}_F)$, and thus, $\tilde{\alpha}_F$ serves as a profitable deviation. Contradiction. $\square$

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