Reputation and Misreporting in News Media

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Abstract

We study the reporting behavior of a reputation-driven news firm. A sender (news firm) dynamically learns about a state via conclusive signals and decides when to make a report to a receiver (consumer). The sender strategically reports to maximize their reputation for learning. In equilibrium, the sender's reputation suffers from delaying reporting and benefits from accuracy. Thus, reputational motives lead to an endogenous speed-accuracy tradeoff. A sender with low ability to learn always *fakes*, i.e. reports despite being ignorant of the state, with positive probability. However, this faking probability decreases over time, and thus later news reports are more trustworthy.

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1. Introduction

The recent addition of social media and non-traditional outlets to the news media lanscape has brought with it concerns about misinformation. However, factual errors have often been a characteristic of traditional media as well. Many such errors by traditional media have regarded some of the most crucial news stories in the United States: the 2000 presidential election [1], the 2013 Boston bombings [2], the Sandy Hook massacre [3], the $9/11$ attacks, the John F. Kennedy assassination [4], etc. The ubiquity of such errors is reflected in the beliefs of consumers: in a Pew Research Center survey from 2009, only 29% of respondents said that news organizations often "get the facts straight", with 63% of respondents expressing a belief that news stories are "often inaccurate".[5]

In this paper, we consider how reputational concerns impact news firms' reporting behavior, and in particular the incidence of misreports. This approach is driven by the observation that news firms' vitality relies heavily on their reputation. A news firm with a reputation for both skilled reporting and journalistic integrity is able to attract and retain consumers who value these attributes. While reputation may matter in a variety of industries, it is particularly salient in the news industry, where given the frequency of exchange between firms and consumers, sustaining these interactions is critical to the firm's welfare.

To this end, we present a model of a reputationally concerned sender, who dynamically learns about an unknown state, and reports to a consumer. Firms wish to maximize their reputation for being "good", which entails both a high ability for learning and a sense of integrity (i.e., honest reporting). We further incorporate a key feature of the news environment: senders decide not only what to report, but when to report.

Our analysis gives rise to three key findings. First, in equilibrium, senders "fake" with positive probability, which entails reporting despite being completely uninformed about the state. This behavior is responsible for a higher incidence of misreporting than that which would prevail if the sender were truthful, i.e., only reporting when she is informed. Importantly, the equilibrium is such that the uninformed sender is indifferent between faking and truth-telling. Notably, we find that faking, and the resulting misreporting, is ceaseless in equilibrium, i.e, it occurs with positive probability at any given time.

[1] Howard Kurtz. *Washington Post*, December 22, 2000.

[2] David Carr. *The New York Times*, April 21, 2013.

[3] Paul Farhi. *Washington Post*, December 18, 2012

[4] Rebecca Greenfield. *The Atlantic*, September 16, 2013

[5] "Press Accuracy Rating Hits Two Decade Low." Pew Research Center, Washington, D.C. (September 13, 2009) https://www.pewresearch.org/politics/2009/09/13/about-the-survey-373/.

Second, we find that in equilibrium, a sender's reputation is rewarded for two separate characteristics of her report: speed and accuracy. That is, while the sender is intrinsically interested solely in maximizing her reputation, she behaves *as if* she wishes to maximize some combination of speed and accuracy. Furthermore, for the sender who is ignorant about the state, these two objectives are at odds, and thus she faces a speed accuracy tradeoff: while faking will prevent her from incurring the reputational deterioration that comes with further delaying a report, truth telling saves her from incurring the reputational harm from making a mistake. In equilibrium, the desire for speed and accuracy precisely counterbalance each other in a way that preserves the uninformed sender's aforementioned indifference between faking and truth telling.

This result is compelling for a number of reasons. With regards to our application, it shows that reputational concerns alone may be responsible for the all-familiar speed-accuracy tradeoff faced by newsrooms. Rather than explicitly modeling this tradeoff, we have provided a microfoundation for it, by showing that it arises endogenously when news media are reputation-concerned. Beyond our application, this result demonstrates speed and accuracy, which are canonically assumed to be of intrinsic value to an individual decision maker, may be of *signalling* value in a delegated learning setting. Finally, because the importance of speed and accuracy is endogenous, this allows us an opportunity to understand *how* they dynamically impact the sender's reputation. With regards to accuracy, we find that while making an accurate report causes an improvement in the sender's reputation and making an inaccurate report causes a decline, the magnitudes of these changes are not equal: the reputational harm from inaccuracy strictly exceeds the reputational gain from accuracy. The interpretation of this result in a news media setting is intuitive: while accurate reporting may cause a modest improvement in a news firm's reputatability, the reputational harm from misreporting is much more consequential.

Thirdly, we consider dynamics in the sender's reporting behavior, specifically how her propensity for misreporting changes over time. We find that if the sender is of sufficently low ability, she becomes strictly *more truthful* as time passes, and thus becomes less likely to misreport. This is despite the fact that speed continues to benefit her reputationally over the course of the game. With regards to our application, this result implies that a news firm is more hasty with its reporting, and thus more likely to misreport, when her research process is still in its early stages, but becomes more scrupulous as time passes.

This paper exists at the intersection of two separate literatures: reputation for learning and games of timing, and specifically those in which decision timing is of signaling value. In the literature on reputation for learning, senders wish to maximize the receivers' belief about their learning ability. Ottaviani and Sørensen (2006b) study this problem in a general static setting, finding that it is generically impossible for senders to truthfully report their information. In other more specific environments (Ottaviani and Sørensen (2006c), Ottaviani and Sørensen (2006a), Dasgupta and Prat (2008), Gentzkow and Shapiro (2006), Prendergast and Stole (1996)), including dynamic ones, this deviation from truth-telling takes various forms, but universally involves low-ability senders manipulating their messages or actions to mimic the behavior of high-ability senders. Two of these papers, Prendergast and Stole (1996) and Dasgupta and Prat (2008), consider dynamic settings. Dasgupta and Prat (2008) model information transmission in financial markets, finding that the degradation of information driven by reputational concerns makes it impossible for prices to converge to the market value. Meanwhile, Prendergast and Stole (1996) model a reputation-concerned investment manager. They find low-ability senders exaggerate their information in early periods, while discounting newer information in later periods.This result stems from the fact that high-ability senders learn more quickly, and thus act more decisively on their information early in the game. As we will illustrate below, this same force is at play in our model: low-ability senders endogenously quicken their reports to partially mimic the high-ability senders' ability to learn faster. Relatedly, Scharfstein and Stein (1990) consider an investment setting, finding that low-ability individuals will act suboptimally due to reputational concerns, where in their setting, consists of herding on the decisions of prior investors. While these latter papers consider dynamic settings, they are not games of timing, i.e., players choose how to act, but not when to do so. This is a vital feature of the media application we consider, as newsrooms decide both what and when to report.

There is an extensive literature on decision timing in games. What is relevent to our work, however, is specifically that in which decision timing is of signalling value (Guttman, Kremer, and Skrzypacz (2014), Gratton, Holden, and Kolotilin (2018), Bobtcheff and Levy (2017), Halac and Kremer (2020)). These papers also come to varying conclusions regarding whether early or later timing is favorable for the sender. In Guttman et al. (2014), managers decide whether and when to disclose information about their firm to the market, and they obtain that later disclosures are viewed more favorably by the market, and thus of greater signalling value to the firm. Meanwhile, Bobtcheff and Levy (2017) consider a cash-constrained firm that endogenously chooses when to make an investment, which in turn serves as a signal to other investors regarding the project's viability. They come to the opposite conclusion: early investment by the firm serves as a signal of project viability, which in turn will increase the firm's ability to obtain additional investment for its project. We obtain a similar outcome to Bobtcheff and Levy (2017), in that in the equilibrium of our model, early reporting is reputationally advantageous for the sender. Notably, however, while in their setting this incentive to act early is driven by a desire to influence the behavior of investors, in our setting, it is purely reputational.

As noted, our paper lies at the intersection of these two literatures. This intersection is also explored by Smirnov and Starkov (2024). Specifically, they model a sender who is concerned with both their interim reputation throughout the game (i.e., at any given time) as well as their terminal reputation. The sender in their model faces a tradeoff between these two types of reputation, which results in dynamics that are qualitatively the opposite of what we obtain: reputation increases conditional on no report, with reports causing instantaneous drops in reputation. Furthermore, this tension between an agent's terminal and interim reputations gives rise to potential non-existence of equilibria, whereas I find that equilibria always exist. I further incorporate learning on the part of the bad agent, and thus find that these agents face a speed-accuracy tradeoff in equilbrium.

The remainder of this paper will proceed as follows. In section 2, we present a dynamic model of a reputation-concerned news firm. In section 3, we characterize the equilibrium in a static version of the model presented in section 2. In section 4, we provide an equilibrium characterization for the full, dynamic model, by building on the static characterization. In section 5, we analyze the equilibrium reputation function, showing that it rewards the sender for both speed and accuracy in reporting. In section 6, we examine dynamics in the sender's reporting behavior, showing that if she is of sufficiently low ability, she becomes more truthful as time passes. Finally, section 7 concludes. All formal proofs are relegated to the Appendix.

2. Model

I present a model of a reputation-driven sender who dynamically learns about an unknown state and chooses when to make a report.

2.1. The game

There is one sender and one receiver. A binary, time-invariant state $\theta \in \{0, 1\}$ is initially unknown to both players. We assume that at the start of the game, both sender and receiver hold prior $Pr(\theta = 1) = \frac{1}{2}$.

The sender has access to a private signal about the state, the informativeness of which depends on her type. The sender's type, denoted by i, may either be "good" ($i = G$) or "bad" ($i = B$). This type is private information: it is known by the sender, but not by the receiver. We let $R_0 \in (0, 1)$ denote the sender's initial reputation, i.e., the receiver's prior belief that the sender is good. The sender learns via conclusive Poisson signals: in any period $t \in \{1, 2, ..., T\}$, θ is privately revealed to the sender with probability λ_i. I.e., in each period, the sender observes a signal r_t:

$$r_t = \begin{cases} \theta & \text{with probability } \lambda_i \\ \emptyset & = \text{with probability } 1 - \lambda_i \end{cases}$$

where $r^t = \emptyset$ indicates that the state was not revealed to the sender at t. Crucially, we assume $\lambda_G > \lambda_B \geq 0$. I.e., the good sender's signal is strictly more (Blackwell) informative than that of the bad sender.

At every period t in which the game has not yet ended, after observing r_t, the sender may choose to *report* to the receiver. Reporting consists of sending a message $m \in \{0, 1\}$ to the receiver. The sender can report at most once over the course of the game, i.e., once she reports, the game ends. Alternatively, the sender can choose to *abstain*, which consists of sending message $m = \emptyset$. If the sender abstains, the game continues to the next period (unless $t = T$, in which case the game ends). The sender is not obligated to report, i.e., she can choose to abstain even in the last period, T. We let τ denote the time at which a report is made, and denote the absence of a report by $\tau = \emptyset$. At $T + 1$ (i.e., after observing the sender's report, or lack thereof) the receiver observes a private signal $s \in \{0, 1\}$ about θ. Specifically, $Pr(s = \theta) = \pi \in (\frac{1}{2}, 1)$. [6] The sender's payoff is given by his reputation, i.e., the receiver's belief that he is the good type, at the end of $T + 1$ (i.e., after the receiver observes τ, m (which equals $\emptyset$ if $\tau = \emptyset$) and s.

2.2. Equilibrium

At any given time, p denotes the sender's belief that $\theta = 1$. A (Markov) strategy for the bad firm at time t is given by $\sigma_t(m, p)$, and for the good firm is given by $\sigma_t^G(m, p)$. This specifies the probability that the sender sends message m under belief p, conditional on not having yet reported. σ_t is a distribution, and thus:

$$\sum_{m \in \{0,1,\emptyset\}} \sigma_t(m, p) = 1.$$

In defining Markov strategies, we are implicitly restricting the strategy of a sender who has learned the state to not depend on time t in which she did so. In equilibrium, this assumption is without loss and is used for notational convenience.

The receiver's beliefs about the sender are denoted by a reputation function R. $R_t(m, s)$

[6] We assume that this private signal is not perfectly correlated with the state to avoid the possibility of off-path occurences where the sender is truthful but their report does not match the state.

denotes the receiver's belief that the sender is good upon observing message $m \in \{0, 1\}$ at time t and private signal s. We let $R(\emptyset, s)$ denote the sender's reputation in the case that she never sends a report.

We seek a perfect Bayesian equilibrium. Specifically, this consists of strategies for each type, σ and σ^G. First, at any given t, all player beliefs (p and R) are consistent with Bayes Rule given the bad (good) sender's strategy σ (σ_G), and in the case of the sender, her private information $r_1, ..., r_t$. Second, the senders' strategies (both type B and type G) must maximize their respective expected reputations at all t and beliefs p they may hold.

I now impose a selection assumption. Specifically, I restrict attention to equilibria where the good sender is truthful: she sends message θ if and only if she knows the state. Formally, she follows strategy σ^G, where for all t,

$$\sigma_t^G(1, 1) = \sigma_t^G(0, 0) = \sigma_t^G(\emptyset, \frac{1}{2}) = 1$$

This assumption is a selection criterion and not a formal restriction on the good sender's behavior. I.e., the good sender is not a commitment type, we are rather restricting attention to equilibria in which the good sender is truthful. This selection criterion also has an economic rationale: we wish to examine equlibria in which news firms' ability is tied to their integrity, i.e., their judiciousness when reporting. In the analysis that follows, we will refer to a perfect Bayesian equilibrium that satisfies the selection assumption as an equilibrium, and we will frequently refer to the bad sender simply as the sender.

3. Static case: characterization

Before analyzing the full dynamic model above, we will provide a characterization for the static case, in which $T = 1$. This will establish certian key results which will extend to the dynamic setting, without having to address the additional analytical complications that dynamics introduce. We show that there is a unique equilibrium. In this equilibrium, a sender who knows the state reports it truthfully, by a sender that does not know the state mixes between reporting truthfully (sending message $\emptyset$) and "faking" (sending message 0 or 1). We will also show that in equilibrium, senders are reputationally rewarded for accuracy. For the remainder of this section, we will drop the time index from all functions.

3.1. Link between strategy and reputation

The below analysis relies heavily on the relationship between the sender's strategy and the reputation function, R. Because the receiver is Bayesian, the reputation function is computed using Bayes Rule as follows:

$$R(m,s) = \frac{1}{1 + L(m,s)\frac{1-R_0}{R_0}},$$

where $L(m,s)$ is the likelihood ratio of outcome (m,s) for bad senders compared to good senders under σ:

$$L(m,s) \equiv \frac{Pr(m,s|B)}{Pr(m,s|G)}.$$ [7]

Thus, the reputation a receiver ascribes to a particular outcome depends on how likely the outcome is for the good sender as compared to the bad sender. This highlights the key force behind our results: the more weight the bad sender's strategy σ places on a particular outcome occuring, the lower the reputation assigned to that outcome will be. In particular, any outcome that is relatively less likely for the bad sender compared to the good sender will cause her reputation to improve compared to her initial reputation R_0, while an outcome that is relatively more likely for the bad sender will cause her reputation to deteriorate.

3.2. Behavior when informed

In this section, we show that the sender truthfully reports arrivals. I.e., she reports 1 (0) if she has learned that the state is 1 (0). To this end, we begin by establishing two lemmas, pertaining to the sender's strategy and the reputation function, respectively.

We begin by establishing a fundamental difference in reporting behavior between the good sender and bad sender. While a good sender reports 0 (1) only if she has learned that $\theta = 1$ ($\theta = 0$), respectively, this is not the case for the bad sender. I.e., the bad sender's reports are not always truthful: she will, with strictly positive probability, send message 1 (0) even when $\theta \neq 1$ ($\theta \neq 0$).

Lemma 1. *In any equilibrium, for $\theta \in \{0,1\}$*

$$\sum_{p \neq \theta} \sigma(\theta, p) > 0.$$

To see why this must hold suppose that the bad sender reports 1 only if she has learned

[7] Formally, $Pr(m,s|i)$ is a function of the sender's strategy, σ^i, e.g.:

$$Pr(m,s|B) = \frac{1}{2}\sum_{\theta\in\{0,1\}}[\sigma^B(m,\theta)\lambda_B + \sigma^B(m,\frac{1}{2})(1-\lambda_B)][\pi + (1-2\pi)\mathbb{I}(\theta \neq s)].$$

that $\theta = 1$. Because the bad type learns that $\theta = 1$ with strictly lower probability, this means that reporting 1 is relatively more likely for the good type. Furthermore, this holds regardless of the receiver's private signal. Thus, a report of 1 guarantees a strictly improved reputation for the sender. Thus, message 1 must serve as a profitable deviation at some p.

Next, we establish a property of the reputation function: the sender's reputation is rewarded for being accurate, namely, for matching the receiver's private signal.

Lemma 2. *In any equilibrium, for $m \neq m' \in \{0,1\}$*

$$R(m,m) > R(m,m').$$

Again, this result is driven by the connection between the sender's strategy and the reputation function. Showing that the sender is rewarded for being accurate is equivalent to showing that that conditional on reporting, a good sender is more likely to be accurate than the bad sender. Because the good type's reports are strictly truthful, they are maximally correlated with s. However, this is not the case for the bad type: by Lemma 1, the sender will report $m \in \{0,1\}$ with strictly positive probability even when the state is not m, and her reports are consequently less correlated with s. It follows that reports by good senders are more likely to be accurate, and thus the reputation function must rewards accuracy.

With these two observations we can now show that the sender must truthfully report the state if she has learned it.

Proposition 1. *In any equilibrium, for $\theta \in \{0,1\}$,*

$$\sigma(\theta,\theta) = 1.$$

Before proceeding, let us discuss the intuition behind this. Consider a sender who has learned that $\theta = 1$ and must decide which message (1, 0, or $\emptyset$) to send. I first argue that the message 0 can't be sent with positive probability. If this were the case, since by Lemma 2 accuracy is reputationally beneficial, message 0 must provide a strictly higher payoff to the sender than 1. I.e., message 1 is sent with zero probability under $p = 0$ and $p = \frac{1}{2}$, a contradiction of Lemma 1. Next, I argue that message $\emptyset$ can't be sent with positive probability. Because the good type is committed to truthful reporting, sending an accurate report signals that the sender knows the state, whereas abstaining signals ignorance. Because the bad type is less likely to know the state than the good type, an informed report makes her appear relatively more reputable than abstaining. For this reason, $\emptyset$ can't be sent with positive proabiblity, either.

3.3. Behavior when uninformed

Proposition 1 tells us that in equilibrium, the bad sender mimics the good sender's strategy when she is informed. I.e., she reports truthfully. Below, we will demonstrate that under non-arrival, this is not the case. In particular, while the good sender abstains (i.e., reports $\emptyset$) when she is uninformed, the bad sender mixes between *faking*, i.e., reporting despite not knowing the state, and abstaining. Furthermore, we show that she fakes the two messages $m \in \{0, 1\}$ with equal probability.

Proposition 2. *In any equilibrium, for all $m \in \{0, 1, \emptyset\}$,*

$$\sigma(m, \frac{1}{2}) > 0.$$

Furthermore, $\sigma(1, \frac{1}{2}) = \sigma(0, \frac{1}{2})$.

To see why the sender must report both 0 and 1 with strictly positive probability when uninformed, recall that by Proposition 1 above, the sender cannot misreport arrivals (i.e., $\sigma(1, 0) = \sigma(0, 1) = 0$). Thus, in order to satisfy Lemma 1, the sender must be reporting both 0 and 1 with strictly positive probability when $p = \frac{1}{2}$. To see why the sender must report $\emptyset$ with positive probability when uninformed, let's consider what would transpire if she didn't. Proposition 1 would then imply that the bad sender never abstains, regardless of her information. Because the good sender does so with strictly positive probability, abstaining would then guarantee a perfect reputation (i.e., $R(\emptyset, 1) = R(\emptyset, 0) = 1$). Abstaining would then serve as a profitable deviation for the bad sender, a contradiction.

Next, let's consider the second part of the proposition, which states that the uninformed sender must mix equally betwen 0 and 1. To see why this must be true, suppose by contradiction that the bad type reports one message $m \in \{0, 1\}$ with greater probability than the other, m'. Because the good type reports the two messages with equal probability, m' is more likely than m to have come from the good type. This means that m' will on average yield a strictly higher reputation for the uninformed sender than m. This in turn implies that m' will serves as a profitable deviation from m when she is uninformed, violating our result that the sender must be indifferent between the two messages.

3.4. Equilibrium existence and uniqueness

Thus far, we have shown that in any equilibrium, the sender truthfully reports arrivals, and when she experiences non-arrival, mixes nontrivally between all three potential messages 0, 1, and $\emptyset$. We claim that there exists a unique strategy in this class that constitutes an equilibrium.

Formally, let σ^b for $b \in (0,1)$ denote the strategy such that

$$\sigma^b(1,1) = \sigma^b(0,0) = 1 \text{ and } \sigma^b(1,\frac{1}{2}) = \sigma^b(0,\frac{1}{2}) = b/2$$

Proposition 3. *There exists a unique equilibrium, consisting of a strategy σ^{b^*}, where $b^* \in (0,1)$*

To see why theree xists a unique equilibrium, consider what happens to the reputation function, and consequently the sender's value function, as b changes. Specifically, let $R^b(m,s)$ denote the reputation function under strategy σ^b, and $V^b(m,p)$ the sender's value from sending message m under belief p. Note that this value function is a function of the reputation function:

$$V^b(m,p) = \tilde{p}R^b(m,1) + (1-\tilde{p})R^b(m,0),$$

where $\tilde{p} \equiv p\pi + (1-p)(1-\pi)$ denotes the probability that $s = 1$ is realized, given belief p about the state. Note that in order for σ^b to constitute an equilibrium, by Proposition 2, it must be that

$$V^b(1,\frac{1}{2}) = V^b(0,\frac{1}{2}) = V^b(\emptyset,\frac{1}{2}).$$

To show that there exists a unique b such that the value functions satisfy this conditions, we make two observations. First, we consider the value function at the two extreme cases of b. When $b = 0$, i.e., when the bad sender is truth-telling, her lower arrival rate compared to the good sender means that reporting will always strictly improve the sender's reputation, whereas abstaining will always harm the sender's reputation. Thus, reporting yields a strictly higher value for the uninformed sender:

$$V^b(1,\frac{1}{2}) = V^b(0,\frac{1}{2}) > V^b(\emptyset,\frac{1}{2}).$$

At the other extreme, when $b = 1$, the bad sender always reports and never abstains, regardless of her belief. Because the good sender abstains with strictly positive probability, this implies that abstaining will guarantee a perfect reputation. So in this case, abstaining will yield a strictly higher value for the uninformed sender:

$$V^b(1,\frac{1}{2}) = V^b(0,\frac{1}{2}) < V^b(\emptyset,\frac{1}{2}).$$

Next, let's consider what happens to the value functions as b increases. Note that as b increases, the bad sender is on average reporting more and abstaining less. Thus, the reputation function will respond by assigning increasingly higher reputation to senders who abstain, and lower reputation to senders who report. This will in turn be reflected in the value functions: the value of reporting is strictly decreasing in b, whereas the value of

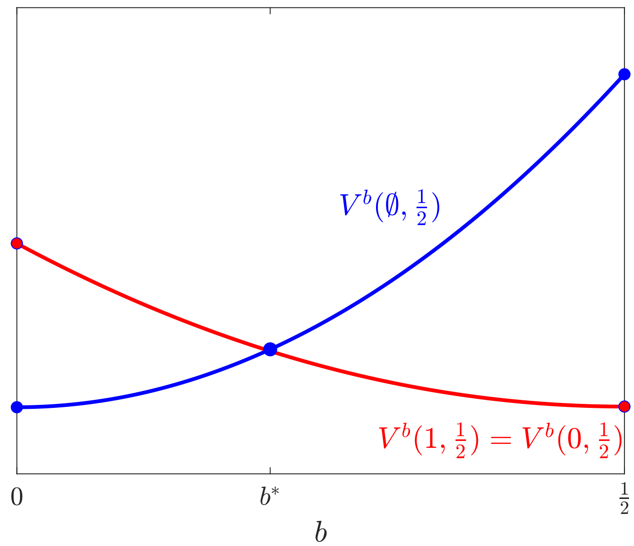

Figure 1: Value function associated with σ^b. b^* denotes unique value such that indifference condition is satisfied.

abstaining is strictly increasing in b (see Figure 1). Given our observations about V^b at the endpoints, this implies that there exists a *unique* point $b^* \in (0, 1)$ such that indifference is achieved.

3.5. Discussion

To summarize, we have shown that the static equilibrium is one in which the bad sender truthfully reports arrivals, but when she does not know the state, she mixes between abstaining and "faking", i.e., making an uninformed report. Furthermore, this equilibrium is one in which senders are endogenously rewarded for making accurate reports (Lemma 2).

This reward for accuracy is crucial to sustaining the uninformed sender's indifference between reporting and abstaining. Because good types are better learners and thus more likely to make informed reports, senders are rewarded for reporting. However, the sender is also penalized for making an inaccurate report (an outcome which is relatively more likely for the uninformed sender), which deters the sender from reporting. In equilibrium, the reputation function is such that these incentives precisely counteract each other when the sender is uninformed, yielding indifference.

As we will discuss below, these same incentives and reporting behaviors prevail under the equilibrium of the dynamic model, in particular the reputation function's endogenous reward for accuracy. The dynamic model, however, further sheds light on how the passage of time impacts both the sender's behavior and her reputation. Understanding these dynamics will be the focus of the sections that follow.

4. Dynamic model: Faking in equilibrium

Above, we showed that in any equilibrium, the sender truthfully reports arrivals, and when she experiences non-arrival, mixes between all three potential messages 0, 1, and $\emptyset$. In this section, we show that the equilibrium in the dynamic setting must also take this form.

4.1. Correlation between state and type

Showing that the dynamic characterization takes the same qualitative form as in the static setting presents unique challenges. One of these involves potential correlation between the state θ and the sender's type i. In the static model, we relied on the assumption that they were uncorrelated, and this assumption was responsible for the resulting symmetry of the sender's strategy across messages (for instance, that $\sigma(1, \frac{1}{2}) = \sigma(0, \frac{1}{2})$). However, one cannot assume this to be true in a dynamic setting. This is due to the fact that the sender's strategy may *induce correlation* between type and state in periods beyond the first. Such correlation will in turn induce behavior in the subsequent periods that violates this symmetry across messages that we obtained in the static setting. In this subsection, we demonstrate that although possible in general, such correlation between the state and the type can never arise in equilibrium.

To this end, we begin by formalizing a notion we refer to as *silence symmetry*.

Definition 1. A strategy is silence symmetric if at time t

$$\sigma_t(\emptyset, 1) = \sigma_t(\emptyset, 0)$$

In words, this condition stipulates that the sender if the sender chooses to withhold a report despite knowing the state (i.e., remaining silent), she must be equally likely to do so in each state. We are concerned with silence symmetry because correlation between the sender's type and the state arises when it is violated.

To fix ideas, let's consider examples of strategies which both satisfy and fail this condition. A simple example of silence symmetry is the case in which the sender is truthful: regardless of the state, an informed sender withholds a report with probability 0. Another example of silence symmetry is one in which the sender never reports in a given period, regardless of her information. In this case, she withholds a report with probability 1, regardless of the state.

Let us now consider an example of a strategy in which silence symmetry is violated. Consider the strategy $\hat{\sigma}_t$ in which the sender truthfully reports 1, but abstains otherwise, i.e.,

$$\hat{\sigma}_t(1,1) = 1, \hat{\sigma}_t(\emptyset,0) = \hat{\sigma}_t(\emptyset,\frac{1}{2}) = 1 \tag{1}$$

In this extreme example of silence asymmetry, the sender *always* withholds a report when she knows the state is 0, but *never* does so when she knows the state is 1. To understand why the failure of silence symmetry can cause correlation between the sender's type and the state, consider the case where $T = 2$ and the bad sender plays $\hat{\sigma}_1$ in the first period. This implies that the bad sender is more likely to survive into the second period (i.e., not report in the first period) under $\theta = 0$. Specifically, when $\theta = 0$, she survives with probability 1, but if the state is 0, she only survives with probability $1 - \lambda_B$. However, this is not the case for the good sender: because she is truthful in equilibrium, she is equally likely to reach period 2 regardless of the state, specifically with probability $1 - \lambda_G$. This in turn implies that at the beginning of the second period, there exists a correlation between the sender's type and the state: $\theta = 0$ with greater probability if the sender is bad than if the sender is good.

4.2. Strategy in equilibrium

Having illustrated the importance of silence symmetry, we begin by establishing a set of necessary conditions that must hold in any equilibrium of the dynamic setup. Among these conditions is silence symmetry.

Lemma 3. *In equilibrium, at all t,*

1. *σ_t is silence symmetric.*
2. *$\sigma_t(1,0) = \sigma_t(0,1) = 0$.*
3. *$\sigma_t(1,\frac{1}{2}) = \sigma_t(0,\frac{1}{2}) > 0$.*

In addition to silence symmetry, we establish two facts that also held in the static setup: namely, that the sender never reports the "wrong" message when she is informed about the state (point 2) and that the sender "fakes" a report with positive probability in every period (point 3). However, unlike the static setting where we were able to show both these points directly, in the dynamic setting, we must take an inductive approach. For instance, silence symmetry at period t is necessary to show that conditions 2 and 3 hold in $t + 1$. To see why this is the case, recall that a failure of silence symmetry in period t would imply that the state is correlated with the sender's type in $t + 1$. This could make one of the two reports (in particular, the one that was withheld less by the bad sender in t) better for the sender's reputation in $t + 1$, all else equal. This could in turn cause violations of conditions 2 and 3 above.

While these necessary conditions substantially narrow the set of candidate equilibria, it still leaves open the possibility that the sender withholds reports with positive probability in a given period, i.e., that $\sigma_t(\emptyset, 1) = \sigma_t(\emptyset, 0) > 0$. We next rule this out as a possibility, establishing that the sender's equilibrium strategy in every period takes the same form as in the static model.

To this end, we begin by defining an T-period equivalent to the class of strategies we defined in the static setting. For any $b \equiv (b_1, ..., b_T) \in (0,1)^T$, let σ^b denote the strategy such that

$$\sigma_t^b(1,1) = \sigma_t(0,0) = 1 \text{ and } \sigma_t^b(1, \frac{1}{2}) = \sigma_t^b(0, \frac{1}{2}) = b_t/2.$$

This strategy is one in which, in every period, the sender truthfully reports arrivals, and mixes nontrivially between all three messages when she does not, specifically reporting 0 and 1 with equal probability. We now claim that any equilibrium must belong to this class. I further establish existence of such an equilibrium, which follows from the Kakutani fixed point theorem.

Proposition 4. *There exists an equilibrium. Furthermore, in any equilibrium, the sender's strategy is given by σ^b, for some $b \in (0,1)^T$.*

Let us now take stock of this result. First, note that in equilibrium, the sender fakes a report with positive probability in *each* period. I.e., faking never ceases. Second, while the equilibrium strategy here is a T-period extension of the static equilibrium strategy, it is dynamic in nature. Specifically, the probability with which the sender reports when she has not learned the state, b_t changes over time. We will explore the dynamics in b_t later in the paper. However, we will begin by examining the equilibrium reputation function, and the dynamics it entails.

5. Reputation: accuracy and speed

Here, we consider how reputation is assessed dynamically in equilibrium. We begin by decomposing the sender's reputation into two components: one capturing the impact of time's passage on her reputation, the other denoting the impact of the report itself, and its associated accuracy, on her reputation. First, we establish that, as in the static setting, the sender is rewarded for accuracy. We specifically show that making an accurate report will cause the sender's reputation to strictly increase from where it was before she reported, whereas an inaccurate report will cause her reputation to strictly decrease. We further examine the relative magnitudes of these changes, and find that the reputational decline that comes from making an inaccurate report strictly exceeds the reputational gain from making an accurate one. Finally, we find that the sender's reputation is rewarded for speed: with every time increment that passes without the sender making a report, her reputation deteriorates.

In order to facilitate the aforementioned decomposition of the sender's reputation function, we begin by defining R_t for $t \in \{0, 1, ..., T\}$, the sender's *interim reputation*. This denotes the receiver's belief about the sender's type conditional on the event that she has not reported at any $s \leq t$. When $t = 0$, R_0 is the sender's prior reputation. For all $t \geq 1$, R_t is an equilibrium object, and can be computed recursively using Bayes' Rule, given the sender's strategy. Under the equilibrium characterization we obtain above, this recursive definition takes the following form:

$$R_t = \frac{1}{1 + \frac{1-R_{t-1}}{R_{t-1}} \frac{(1-\lambda_B)(1-b_t)}{1-\lambda_G}}.$$

Now let us interpret this object. While in our model the sender is solely concerned with her reputation at the end of the game, R_t captures how the sender's reputation dynamically evolves as the game progresses. Specifically, R_t specifies the sender's reputation at periods preceding that in which she reports. For instance, if the sender were to report in period $t = 5$, R_4 denotes where her reputation stood immediately before her report was made.

By linearly separating the sender's interim reputation from her reputation function, it can be written in the following form:

$$R_t(m, s) = R_{t-1} + \alpha_t \mathbb{I}(m = s) + \beta_t \mathbb{I}(m \neq s), \tag{2}$$

where, like R_t, α_t and β_t are equilibrium objects and functions of the sender's equilibrium strategy. Note that (2) implicitly assumes that the sender's reputation depends on the content of her report (whether $m \in \{0, 1\}$) only to the extent that it impacts her accuracy (i.e., whether $m = s$). While we have omitted a formal proof of this, the reputation function can be written in this way due to the fact that in equilibrium, the sender's strategy exhibits symmetry across messages $m \in \{0, 1\}$ and states $\theta \in \{0, 1\}$ in every period.

Now, let us interpret this decomposition. As discussed above, the first component, R_{t-1}, denotes the sender's reputation immediately prior to her time t report. This component thus captures all the dynamic change in her reputatation that happened prior to her report, and specifically, the impact that *delay* had on her reputation. Meanwhile, the residual component, $\alpha_t \mathbb{I}(m = s) + \beta_t \mathbb{I}(m \neq s)$, denotes the change that occurs in the sender's reputation at the moment in which she reports. This component accounts for the impact that accuracy has on the sender's reputation. Note that the α_t and β_t are time-contingent: this is due to the fact that the magnitude of impact accurate (or inaccurate) reporting has on the sender's reputation, like R_t, is dynamic in nature. Figure 2 below illustrates how the sender's reputation evolves dynamically, and sheds further light on this decomposition: the dotted lines denote time paths of the sender's reputation, when she reports at $t = 5$, either accurately (blue line) or inaccurately (red line). Her reputation at all periods prior to her report are given by R_t, and the final jump in her reputation that occurs in the period in which she reports is given by α_t or β_t, if her report was accurate or inaccurate, respectively.

5.1. Accuracy

With this decomposition in hand, we first seek to understand the role that accuracy plays in the sender's reputation. Our findings are summarized by the following proposition.

Proposition 5 (Accuracy and reputation)**.** *In any equilibrium*

1. $\alpha_t > 0$ *and* $\beta_t < 0$.
2. $-\beta_t > \alpha_t$.

Proof. See appendix. □

This proposition makes two separate claims. Let us begin by understanding the first, i.e., that $\alpha_t > 0$ and $\beta_t < 0$ for all t. This claim asserts that at any given point in time, a correct report will cause an increase in the sender's reputation compared to where it stood immediately prior to the report, whereas an incorrect report will cause a decrease. In part, this is an extension of Lemma 2 from the static model, in that it implies that in any period, a sender's reputation from making a correct report, $R_t(1, 1)$, strictly exceeds her reputation from making an incorrect report, $R_t(1, 0)$. As in the static model, this is driven by the fact that the bad sender fakes with strictly positive probability in any given period, while the good sender does not. Because faking is associated with inaccurate reporting, inaccuracy is thus reputation-damaging.

However, this claim goes a step further than this: it additionally asserts that the sender's reputation following a correct report must strictly exceed her interim reputation immediately prior to that report, whereas her reputation following an incorrect report must lie strictly below her interim reputation. That is, accurate reporting causes an immediate increase in the sender's reputation, whereas inaccurate reporting causes a deterioration. The intuition for this is clear: suppose by contradiction that both α_t and β_t were positive. Then, all time-t senders could guarantee an improvement in their reputations by reporting. It would thus follow that at the end of the game, the sender's reputation will improve with probability 1, regardless of her ability. Thus, the reputation function must not be consistent with Bayes

Rule, a contradiction. If, instead we assume that both α_t and β_t were negative, no sender would choose to report at time t, as it will cause her reputation to decline with probability 1, even if her report is accurate. Instead, she would elect to abstain, in which case her reputation would evolve to R_t, which must, in order to be consistent with a negative α_t and β_t, lie above R_{t-1}.

Let us now consider the second component of the claim: namely, that $-\beta_t > \alpha_t$. This claim asserts that the reputational deterioration resulting from an inaccurate report strictly exceeds the reputational growth that occurs with an accurate report. I.e., the sender has more to lose from an erroneous report than she has to gain from a correct one. Consequently, if an uninformed sender chooses to fake a report, she is choosing to take a binary gamble where the cost incurred when losing (i.e., making an incorrect report) outweigh the benefit earned when winning (i.e., making a correct report).

This result is due to the features of both the sender's strategy and the reputation function in equilibrium. We begin by observing that the equilibrium is one that is strictly informative about the sender's type, i.e., the reputation function is not constant. It follows that reporting behavior exhibited more often by the bad type will cause her reputation to decline on average. This holds in particular when the sender *fakes*, as it is done with positive probability by the bad sender, but never by the good one. Because the uninformed sender holds a belief $\frac{1}{2}$ about the states, a sender who fakes will be accurate half of the time. Thus, her expected reputation from faking at time t is given by

$$R_{t-1} + \frac{\alpha_t - \beta_t}{2}$$

In order to ensure that the faking sender's reputation declines, it must be that $\alpha_t < -\beta_t$. Intuitively, the sender must be penalized for faking, otherwise, she would be guaranteed to sustain a reputation of at least R_{t-1} by the end of the period t, which is not possible in any equilibrium that is informative about the sender's type.

5.2. Speed

Next, we consider how the sender's timing affects her reputation. Our key claim is the following, which asserts that delaying reporting strictly damages the sender's reputation.

Proposition 6 (Speed and reputation)**.** *In equilibrium, R_t is strictly decreasing in t.*

The proof for this claim follows a backwards induction argument. We begin by arguing that $R_T < R_{T-1}$. To see why this must hold, note that $R_T = R_T(\emptyset, 1) = R_T(\emptyset, 0)$, i.e., R_T equals the reputation the sender enjoys in the event that she stays silent through the course

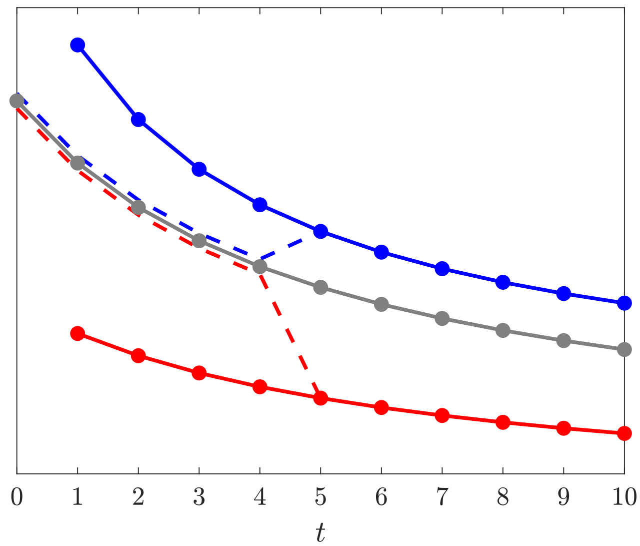

Figure 2: Dynamics of the sender's reputation function. The grey line denotes R_t, solid red (blue) line denotes reputation from making an accurate (inaccurate report) at t. The dotted red (blue) line shows dynamics of reputation for a sender who makes an inaccurate (accurate) report in period 5.

of the entire game. If we assume by contradiction that $R_T \geq R_{T-1}$, abstaining at time T when uninformed serves as a profitable deviation: as we demonstrated above, this is due to the fact that faking at time T yields a reputation strictly less than R_{T-1}.

Next, let's consider an arbitrary period t, and assume by contradiction that $R_t \geq R_{t-1}$. Unlike T, it does not directly follow that uninformed sender can profitably deviate by abstaining at t. This is because the game does not end at $t+1$, and her reputation may decline following this. In particular, even if the sender starts period $t+1$ with a relatively high reputation R_t, if b_{t+1} is sufficiently high, her value from reporting will be relatively low. This is because the more the bad sender reports on average (which is associated with a higher b_{t+1}), the worse reporting in $t+1$ will be for her reputation, regardless of her accuracy. Thus, in order to show that the sender can profitably deviate by abstaining in period t when she is uninformed, we must show that b_{t+1} remains relatively low.

As we will now show, this follows directly from the inductive assumption that $R_t > R_{t+1}$. Specifically, $R_t > R_{t+1}$ implies that b_{t+1} lies below some threshold $\bar{b}$. [8]. It is the unique value of b_t such that the good and bad sender abstain with equal probability, i.e., it is the solution to the following equality: $1 - \lambda_G = (1 - \lambda_B)(1 - b)$. There is a clear intuition for this: the more the bad sender fakes (i.e., the higher b_{t+1} is), the *better* abstaining must be for the sender's reputation, as it is less probable for the bad sender. This in turn results in

[8] Formally, $\bar{b} \equiv \frac{\lambda_G - \lambda_B}{1 - \lambda_B}$

a higher R_{t+1}. Thus, b_{t+1} must lie below the above threshold to ensure that R_{t+1} does not exceed R_t. This bound we obtain is sufficient to show that the uninformed sender at t can profitably deviate by abstaining at t, thus showing that we cannot have $R_t \geq R_{t-1}$.

5.3. Discussion

In this section we have shown that the equilibrium reputation function rewards the sender for two qualities of her report: speed and accuracy. Thus, although the sender is reputation-maximizing, she behaves as if she is maximizing some combination of speed and accuracy. Notably, in canonical settings with exogenous payoff functions, it is often assumed that her payoff is an increasing function of speed and accuracy (e.g., Wald (1947)). We have shown that in a reputational setting, the same holds, but arises endogenously. That is, while speed and accuracy may be of intrinsic value in the canonical setting, they are in equilibrium of signaling value in a reputational setting.

The dynamics of the sender's reputation, as well as the importance of both speed and accuracy, are shown in Figure 2, which illustrates our findings from this section. As shown, the sender's reputation is dynamically discounted until the time in which she reports (Proposition 6). Once she reports, her reputation will exhibit a strict upwards jump if her report was accurate, and a downwards jump if it was inaccurate (Proposition 5). Also in line with Proposition 5, we see that the loss in reputation the sender sustains from making an inaccurate report exceeds the gain she enjoys when she makes an accurate one.

6. Dynamics in strategy

In this section, we examine dynamics in the sender's strategy. We show that b_t is strictly decreasing in t, provided that λ_B lies below some bound.

Proposition 7 (Declining b_t)**.** *In equilibrium, there exists a $\underline{\lambda} \in (0, \lambda_G)$ such that if $\lambda_B < \underline{\lambda}$, b_t is strictly decreasing in t.*

To understand why this result holds, first recall Proposition 6, which tells us that conditional on not having reported, the sender's reputation at the beginning of period $t+1$, R_t, must be strictly less than her reputation at the beginning of period t, R_{t-1}. This means that all else equal, and in particular if the bad sender were to employ the same strategy in both periods (i.e., if $b_t = b_{t+1}$), reporting at time t provides the sender with a higher reputation than reporting at time $t+1$ regardless of whether or not the report was accurate. For the sender who is uninformed about the state at t, this makes faking at time t particularly attractive, as it ensures that she will enjoy a higher reputation on average than she would by faking at time $t + 1$ instead.

Thus, in order to ensure that the uninformed sender at t does not have a strict incentive fake at time t, thus violating the indifference condition, she must be compensated for waiting through two separate channels. First, by waiting, she will with positive probability learn the state in the next period, and increase her chances of making an accurate report. If however this probability (λ_B) is sufficiently small, she must instead be compensated through a different channel: the reputation function. In particular, although the sender starts off $t + 1$ with a lower reputation, she must exhibit a greater boost in reputation from actually reporting. To see why this implies that $b_t > b_{t+1}$, consider the relationship between b_t and the reputation function. Because a higher b_t is associated with a higher probability that the bad sender will report, the reputation function will respond by assigning a *lower* reputation to reporting at time t (regardless of the sender's accuracy). Thus, a greater increase in reputation from reporting at $t + 1$ will happen only if $b_t > b_{t+1}$.

Economically, this result tells us that a sender who is of sufficiently low ability becomes *more truthful* as time passes, i.e., is less likely fake a report. This indicates that the sender's approach to reporting changes as the game progresses. As we have described above, the sender has two separate means of convincing the receiver she is of high ability: the speed of her report, and its accuracy. While both factors are present throughout the course of the game, this result indicates that in earlier periods, the bad sender relies relatively more on the speed approach to convince the receiver she is good, accomplished through a higher b_t. However, as time passes, she gradually shifts her behavior towards the accuracy approach, by trying to convince the receiver she is good through her reluctance for dishonesty.

7. Conclusion

In this paper, we sought to understand the nature of news reporting by a sender who is reputation-maximizing. Our analysis gave rise to several findings, regarding both the sender's reporting behavior as well as her incentives in equilibrium. First, we find that in equilibrium, the sender will fake a report with positive probability in every period, which entails reporting in the absence of knowledge about the state. Next, we find that the sender is endogenously rewarded two separate qualities of their report: speed and accuracy. We thus provide a reputational microfoundation for the speed-accuracy tradeoff in the newsroom setting. We make a further observation about the nature of accuracy's importance: senders are in equilibrium penalized strictly more for inaccurate reporting than they are rewarded for accurate reporting, meaning that erroneous reporting is more consequential for the sender's reputation than accurate reporting. Finally, we explore dynamics in the sender's reporting behavior, finding that if the sender is of sufficiently low ability, she will become strictly more truthful as time passes. This implies that misreports by a sender who has a low a capability for learning are most probable in the immediate aftermath of obtaining a lead, and becomes less probable as time passes.

While our model provides a simple framework for understanding reputation-driven misreporting by news media, some of our findings warrant further investigation in more general frameworks. In particular, it is not clear that the endogenous reputational reward for both speed and accuracy obtained under our model will exist under a general learning structure on the part of the sender, and even relaxing our selection assumption on the good sender's reporting behavior. Furthermore, it is unclear if additional reporting incentives arise in a more general framework, beyond just speed and accuracy. These questions present avenues for further research.

References

Catherine Bobtcheff and Raphaël Levy. More haste, less speed? signaling through investment timing. *American Economic Journal: Microeconomics*, 9(3):148–86, 2017.

Amil Dasgupta and Andrea Prat. Information aggregation in financial markets with career concerns. *Journal of Economic Theory*, 143(1):83–113, 2008.

Matthew Gentzkow and Jesse M Shapiro. Media bias and reputation. *Journal of political Economy*, 114(2):280–316, 2006.

Gabriele Gratton, Richard Holden, and Anton Kolotilin. When to drop a bombshell. *The Review of Economic Studies*, 85(4):2139–2172, 2018.

Ilan Guttman, Ilan Kremer, and Andrzej Skrzypacz. Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review*, 104(8):2400–2420, 2014.

Marina Halac and Ilan Kremer. Experimenting with career concerns. *American Economic Journal: Microeconomics*, 12(1):260–88, 2020.

Marco Ottaviani and Peter Norman Sørensen. Professional advice. *Journal of Economic Theory*, 126(1):120–142, 2006a.

Marco Ottaviani and Peter Norman Sørensen. Reputational cheap talk. *The Rand journal of economics*, 37(1):155–175, 2006b.

Marco Ottaviani and Peter Norman Sørensen. The strategy of professional forecasting. *Journal of Financial Economics*, 81(2):441–466, 2006c.

Canice Prendergast and Lars Stole. Impetuous youngsters and jaded old-timers: Acquiring a reputation for learning. *Journal of political Economy*, 104(6):1105–1134, 1996.

David S Scharfstein and Jeremy C Stein. Herd behavior and investment. *The American economic review*, pages 465–479, 1990.

Aleksei Smirnov and Egor Starkov. Timing of predictions in dynamic cheap talk: experts vs. quacks. *University of Zurich, Department of Economics, Working Paper*, (334), 2024.

Abraham Wald. Foundations of a general theory of sequential decision functions. *Econometrica, Journal of the Econometric Society*, pages 279–313, 1947.

8. Appendix

8.1. Relevant notation and properties

Before proceeding, we will introduce some relevant notation and properties which will prove useful in the analysis which follows.

We begin with the sender's value function. Let $V_t^i(m,p)$ denote type i sender's value from sending message m at time t when she holds belief p. If $m = \emptyset$ this is equal to the sender's continuation value. If $m \in \{0,1\}$, this value is a linear function of the sender's belief p:

$$V_t^i(m,p) = \tilde{p}R(m,1) + (1-\tilde{p})R(m,0)$$

for $m \in \{0,1\}$, where $\tilde{p}$ is the sender's belief $s=1$ will realize, formally $\tilde{p} \equiv p\pi + (1-p)(1-\pi)$. We further let $V_t^i(p)$ denote the sender's value at time t under belief p.

$$V_t(p) \equiv \max_m V_t(m,p).$$

For brevity, in the analysis that follows, we will often drop the i superscript when referring to the bad sender.

Next, we define a function of the sender's strategy σ, which we call the *effective arrival rate*. This denotes the probability with which the sender will know the state is either 0 or 1 at time t, respectively. We now formally define this object:

Definition 2. The effective arrival rate of θ at time t is given by

$$\lambda_B^{t,\theta} = \begin{cases} \frac{\lambda_B}{2} & \text{if } t = 1 \\ (1 - \lambda_B^{t-1,1}\sigma_{t-1}(\emptyset, 1) - \lambda_B^{t-1,0}\sigma_{t-1}(\emptyset, 0))\frac{\lambda_B}{2} + \lambda_B^{t-1,\theta}\sigma_{t-1}(\emptyset, \theta) & \text{if } t > 1 \end{cases}$$

In the case where $\sigma_s(\emptyset, 0) = \sigma_s(\emptyset, 1)$ for all $s < t$, which holds in much of the analysis below, it follows from the above definition that $\lambda_B^{t,0} = \lambda_B^{t,1}$. Thus to simplify notation, we will in these cases remove the state superscript from the effective arrival rate, and let

$$\lambda_B^t \equiv \lambda_B^{t,0} + \lambda_B^{t,1}.$$

Now, we will formally define the sender's interim reputation, which as noted in the main text, denotes the receiver's belief about the sender's type, given that she has not reported on or before t. While we provide a formula in the main text, that formula is only relevant under the equilibrium strategy which we derive. The general formula follows recursively from Bayes Rule and is given by the following for all $t \in \{1, ..., T\}$:

$$R_t = \frac{1}{1 + (\frac{1-R_{t-1}}{R_{t-1}} \frac{\lambda_B^{t,0}\sigma(\emptyset,0)+\lambda_B^{t,1}\sigma(\emptyset,1)+(1-\lambda_B^{t,0}-\lambda_B^{t,1})\sigma(\emptyset,\frac{1}{2})}{1-\lambda_G})}.$$

Meanwhile, the reputation function $R_t(m, s)$ denotes the receiver's belief that the sender is the good type, given that she sends message m and time t, and the receiver observes private signal s. Let E_t denote the event that the sender does not report before time t, i.e., that $\tau \geq t$. Then Bayes Rule yields that for $m \in \{0, 1\}$:

$$R_t(m, s) = \frac{1}{1 + \frac{1-R_{t-1}}{R_{t-1}} L_t(m, s)}.$$

where $L_t(m, s) \equiv \frac{Pr(m,s|B,E_t)}{Pr(m,s|G,E_t)}$, and $Pr(m, s|i, E_t)$ denotes the probability that $(\tau = t, m, s)$ will realize, given that a sender of type i hadn't reported before time t.

Proof of Lemma 1. Here we prove that the claim holds for $\theta = 1$. I a symmetric argument can be used to prove the claim for $\theta = 0$.

Suppose by contradiction that $\sigma(1, 0) = \sigma(1, \frac{1}{2}) = 0$. Let $Pr(m, s|i)$ denote the probability that (m, s) is realized given the sender's type is i. Then by Bayes Rule:

$$Pr(1, 1|B) = \lambda_B\sigma(1, 1)\pi \text{ and } Pr(1, 1|G) = \lambda_G\pi.$$

It follows that

$$L(1,1) = L(1,0) = \frac{\lambda_B \sigma(1,1)}{\lambda_G} < 1.$$

This in turn implies that $V(p) \geq V(1,p) > R_0$, i.e., that the bad sender's reputation will strictly improve with probability 1. Thus, R must be in violation of Bayes Rule: contradiction.

□

Proof of Lemma 2. Here, we prove that $R(1,1) > R(1,0)$. That $R(0,0) > R(0,1)$ follows symmetrically. By the expression for R, showing $R(1,1) > R(1,0)$ is equivalent to showing that $L(1,1) < L(1,0)$. It follows by Bayes Rule that

$$L(1,1) = \frac{1}{\lambda_G}[\frac{1-\pi}{\pi}P(m=1|\theta=0,B) + P(m=1|\theta=1,B)]$$

$$L(1,0) = \frac{1}{\lambda_G}[\frac{\pi}{1-\pi}P(m=1|\theta=0,B) + P(m=1|\theta=1,B)],$$

where $P(m|\theta,B)$ denotes the probability that message m is sent, given the state is θ and the sender is bad. It is given by:

$$Pr(m|\theta,B) = \sigma(m,\theta)\lambda_B + \sigma(m,\frac{1}{2})(1-\lambda_B).$$

It follows from Lemma 1 that $Pr(m=1|\theta=0,B) > 0$. Since by assumption $\pi > \frac{1}{2}$, it follows that $L(1,1) < L(1,0)$. □

We will now prove a corollary of Lemma 2, which will be used in proving Proposition 1. This corollary establishes strict monotonicity of the value functions:

Corollary 1. *In any equilibrium, $V(1,p)$ ($V(0,p)$) is strictly increasing (decreasing) in p.*

Proof of Corollary 1. We will prove that $V(1,p)$ is strictly increasing. That $V(0,p)$ is strictly decreasing follows symmetrically. Rerranging the above formula for $V(1,p)$, we have:

$$V(1,p) = p(2\pi-1)[R(1,1) - R(1,0)] + [R(1,1)(1-\pi) + R(1,0)\pi]$$

By Lemma 2, it follows that $R(1,1) - R(1,0) > 0$, and thus $V(1,p)$ is strictly increasing in p. □

Proof of Proposition 1. We will prove that $\sigma(1,1) = 1$. That $\sigma(0,0) = 1$ follows symmetrically.

We will now show $\sigma(0,1) = \sigma(\emptyset,1) = 0$. First, suppose by contradiction that $\sigma(0,1) > 0$. It follows that message 0 is optimal under $p = 1$, and thus $V(0,1) \geq V(1,1)$. It follows from

Corollary 1 that

$$V(0, p) > V(0, p) \text{ for all } p < 1.$$

Thus, $\sigma(1, \frac{1}{2}) = \sigma(1, 0) = 0$. This is a contradiction of Lemma 1.

Now, suppose by contradiction that $\sigma(\emptyset, 1) > 0$. To this end we first show that $R(\emptyset, 1) > R(\emptyset, 0)$. By the above formula for V, this is equivalent to showing that $V(\emptyset, 1) > V(\emptyset, \frac{1}{2})$, which follows from the following chain of inequalities:

$$V(\emptyset, 1) \geq V(1, 1) > V(1, \frac{1}{2}) \geq V(\emptyset, \frac{1}{2}).$$

The first inequality in this chain follows from the assumption that message $\emptyset$ is optimal under $p = 1$. The second inequality follows from Corollary 1. The third inequality holds because $\sigma(1, 0) = 0$, which implies by Lemma 1 that $\sigma(1, \frac{1}{2}) > 0$.

Next, we show $\sigma(\emptyset, 0) = 0$. It follows from Lemma 1 that $V(\emptyset, \frac{1}{2}) \leq V(0, \frac{1}{2})$. Because, $V(\emptyset, \cdot)$ is strictly increasing while $V(0, \cdot)$ is strictly decreasing, this implies $V(\emptyset, 0) < V(0, 0)$, thus showing that $\sigma(\emptyset, 0) = 0$. This implies:

$$P(m = \emptyset | \theta = 0, B) = (1 - \lambda_B)\sigma(\emptyset, \frac{1}{2}) < \lambda_B \sigma_B(\emptyset, 1) + (1 - \lambda_B)\sigma(\emptyset, \frac{1}{2}) = P(m = \emptyset | \theta = 1, B)$$

which implies:

$$L(\emptyset, 0) = \frac{(1 - \pi)P(\emptyset|1, B) + \pi P(\emptyset|0, B)}{1 - \lambda_G} < \frac{\pi P(\emptyset|1, B) + (1 - \pi)P(\emptyset|0, B)}{1 - \lambda_G} = L(\emptyset, 1).$$

This inequality on L is equivalent to $R(\emptyset, 0) > R(\emptyset, 1)$, which is a contradiction of the above. □

Proof of Proposition 2. We begin by proving the first part of the proposition, namely that $\sigma(m, \frac{1}{2}) > 0$ for all m. First, note that because by Proposition 1 $\sigma(1, 0) = \sigma(0, 1) = 0$, in order to satisfy Lemma 1, it must be that $\sigma(m, \frac{1}{2}) > 0$ for $m \in \{0, 1\}$. Next, suppose by contradiction that $\sigma(\emptyset, \frac{1}{2}) = 0$. It then follows by Lemma 1 that the bad sender never sends message $\emptyset$. Because the good type does so with strictly positive probability, a $\emptyset$ message will reveal that the sender is good, i.e., $R(\emptyset, 1) = R(\emptyset, 0) = 1$. Thus a report of $\emptyset$ serves as a profitable deviation for the bad sender.

Next, we prove the second part of the proposition, namely, that $\sigma(1, \frac{1}{2}) = \sigma(0, \frac{1}{2})$. Assume by contadiction that this does not hold. Without loss of generality, let us assume that

$\sigma(1, \frac{1}{2}) > \sigma(0, \frac{1}{2})$. By definition,

$$L(0,0) = \frac{(1-\pi)(1-\lambda_B)\sigma(0,\frac{1}{2}) + \pi(\lambda_B + (1-\lambda_B)\sigma(0,\frac{1}{2}))}{\pi\lambda_G}$$
$$< \frac{(1-\pi)(1-\lambda_B)\sigma(1,\frac{1}{2}) + \pi(\lambda_B + (1-\lambda_B)\sigma(1,\frac{1}{2}))}{\pi\lambda_G} = L(1,1)$$

Furthermore,

$$L(0,1) = \frac{\pi(1-\lambda_B)\sigma(0,\frac{1}{2}) + (1-\pi)(\lambda_B + (1-\lambda_B)\sigma(0,\frac{1}{2}))}{(1-\pi)\lambda_G}$$
$$< \frac{\pi(1-\lambda_B)\sigma(1,\frac{1}{2}) + (1-\pi)(\lambda_B + (1-\lambda_B)\sigma(1,\frac{1}{2}))}{(1-\pi)\lambda_G} = L(1,0)$$

It follows from these two inequalities that $R(0,0) > R(1,1)$ and $R(0,1) < R(1,0)$. Thus,

$$V(0, \frac{1}{2}) = \frac{1}{2}R(0,0) + \frac{1}{2}R(0,1) > \frac{1}{2}R(1,1) + \frac{1}{2}R(1,0) = V(1, \frac{1}{2})$$

However, this implies that $\sigma(1, \frac{1}{2}) = 0$, which is a contradiction of the first part of the claim. □

Before proceeding with the proof for Proposition 3, we introduce some relevant notation. Recalling the definition of σ^b above, let R^b, V^b, and L^b denote the reputation, value and likelihood functions, respectively that are consistent with σ^b. Furthermore, let

$$X(b) \equiv V^b(1, \frac{1}{2}) - V^b(\emptyset, \frac{1}{2}).$$

Proof of Proposition 3. We wish to show that there exists a unique b such that σ^b constitutes an equilibrium. By Proposition 2, if σ^b constitutes an equilibrium, then $X(b) = 0$. So, we begin by showing that there exists a unique $b^* \in (0,1)$ such that $X(b^*) = 0$. To this end, we make two observations about $X(b)$:

1. $X(b)$ is continuous and strictly decreasing in b. It suffices to show that $V^b(1, \frac{1}{2})$ is continuous and strictly decreasing in b and $V^b(\emptyset, \frac{1}{2})$ is continuous and strictly increasing in b. First note that for all $s \in \{0, 1\}$,

$$L^b(1, s) = \frac{(1-\lambda_B)b/2 + Pr(s|\theta = 1)\lambda_B}{Pr(s|\theta = 1)\lambda_G}$$

which implies that $R^b(1, s)$ is continuous and strictly decreasing in b for $s \in \{0, 1\}$, and thus that $V^b(1, \frac{1}{2})$ is continuous and strictly decreasing in b.

Next, note that

$$L^b(\emptyset, s) = \frac{(1-\lambda_B)(1-b)}{(1-\lambda_G)} \text{ for } s \in \{0,1\}$$

which implies that $R^b(\emptyset, s)$, and consequently $V^b(\emptyset, \frac{1}{2})$ is continuous and strictly increasing in b for $s \in \{0,1\}$.

2. $X(0) > 0$ and $X(1) < 0$. To show $X(0) > 0$, note that by the above formulae, for $s \in \{0,1\}$,

$$L^0(1, s) = \frac{\lambda_B}{\lambda_G} < \frac{1-\lambda_B}{1-\lambda_G} = L^0(\emptyset, s)$$

Thus, $R^0(1, s) > R^0(\emptyset, s)$ for $s \in \{0,1\}$. Therefore, $X(0) > 0$. To show $X(1) < 0$, note that for $s \in \{0,1\}$, $L^1(1, s) > 0 = L^1(\emptyset, s)$. Thus, $R^1(1, s) < R^1(\emptyset, s)$ for all $s \in \{0,1\}$. Thus, $X(1) < 0$.

Combining the above two observations, it follows that there exists a unique $b^* \in (0,1)$ such that $X(b^*) = 0$. Thus we have shown that the only candidate equilibrium is (σ^{b^*}, R^{b^*}). It remains to confirm that this is indeed an equilibrium, i.e., that the sender cannot profitably deviate at any possible belief.

Let us begin with the belief $p = \frac{1}{2}$. First, note that under $m_B^{b^*}$, $V^{b^*}(1, \frac{1}{2}) = V^{b^*}(0, \frac{1}{2})$. Thus, because $X(b^*) = 0$

$$V^{b^*}(1/2) = V^{b^*}(m, 1/2) \text{for all } m. \tag{3}$$

Next, we will show there does not exist a profitable deviation when $p = 1$. That there does not exist a profitable deviation when $p = 0$ follows symmetrically. To show this, first note by definition of L:

- $L^{b^*}(\emptyset, 0) = L^{b^*}(\emptyset, 1)$
- $L^{b^*}(1, 0) > L^{b^*}(1, 1)$
- $L^{b^*}(0, 1) > L^{b^*}(0, 0)$.

It follows from these three inequalities that

- $V^{b^*}(\emptyset, p)$ is constant in p
- $V^{b^*}(1, p)$ is strictly increasing in p
- $V^{b^*}(0, p)$ is strictly decreasing in p.

It follows from (5) that

$$V^{b^*}(1, 1) > V^{b^*}(\emptyset, 1) > V^{b^*}(0, 1).$$

Thus, $m = 1$ is the unique best response at $p = 1$, and there is no profitable deviation. □

8.2. Proofs for dynamic model

We begin by proving a Lemma that will be of use in our analysis below:

Lemma 4. *In any equilibrium, $V_t(1,p)$ ($V_t(0,p)$) is weakly increasing (decreasing) in p. Furthermore, $V_t(1,p)$ ($V_t(0,p)$) is strictly increasing (decreasing) in p whenever $\sigma_t(1,0)+\sigma_t(1,\frac{1}{2}) > 0$ ($\sigma_t(0,1)+\sigma_t(0,\frac{1}{2}) > 0$).*

Proof. We begin by proving that $V_t(1,p)$ is weakly increasing in p. All results for $V_t(0,p)$ follow symmetrically. First recall that

$$V_t(1,p) = p(2\pi-1)[R_t(1,1) - R_t(1,0)] + [R_t(1,1)(1-\pi) + R_t(1,0)\pi]$$

Thus, to show that $V_t(1,p)$ is weakly increasing in p, it suffices to show that $R_t(1,1) \geq R_t(1,0)$. To show this, by definition of R, it suffices to show that $L_t(1,1) \leq L_t(1,0)$. This holds by definition, since

$$L_t(1,1) = \frac{\lambda_B^{t,1}\sigma_t(1,1) + \lambda_B^{t,0}\frac{1-\pi}{\pi}\sigma_t(1,0) + (1-\lambda_B^{t,1}-\lambda_B^{t,0})\frac{\sigma_t(1,\frac{1}{2})}{2\pi}}{\lambda_G/2} \leq$$
$$\frac{\lambda_B^{t,1}\sigma_t(1,1) + \lambda_B^{t,0}\frac{\pi}{1-\pi}\sigma_t(1,0) + (1-\lambda_B^{t,1}-\lambda_B^{t,0})\frac{\sigma_t(1,\frac{1}{2})}{2(1-\pi)}}{\lambda_G/2} = L_t(1,0)$$

Now, suppose $\sigma_t(1,0)+\sigma_t(1,\frac{1}{2}) > 0$. In this case, the above weak inequality on $L_t(1,1)$ and $L_t(1,0)$ will become a strict inequality, thus yielding that $R(t,1,1) > R(t,1,0)$, and thus that $V_t(1,p)$ is strictly increasing in p. □

Proof of Lemma 3. Fix a t. Suppose by induction that all three claims hold for $s < t$ (this holds vacuously when $t = 1$).

We wish to show that $\sigma_t(1,0) = \sigma_t(0,1) = 0$, and that $\sigma_t(m,\frac{1}{2}) > 0$ for $m \in \{0,1\}$. We first consider the case where $\lambda_B^t < \lambda_G$. Let us begin by showing $\sigma_t(1,0) = \sigma_t(0,1) = 0$. Suppose by contradiction that $\sigma_t(0,1) > 0$. This implies by Lemma 4 that $V_t(0,p)$ is strictly decreasing in p. It thus follows that $V_t(0,p) > V_t(1,p)$ for all $p < 1$. This, however, implies that the sender will only ever report 1 when $p = 1$, i.e.,

$$\sigma_t(1,0) = \sigma_t(1,\frac{1}{2}) = 0$$

Thus,

$$L_t(1,1) = L_t(1,0) = \frac{\lambda_B^t\sigma_t(1,1)}{\lambda_G} < 1$$

Since by assumption $\lambda_B^t < \lambda_G$, this implies that $R_t(1,1) = R_t(1,0) > R_{t-1}$, i.e., that the sender can guarantee an improved reputation by reporting 1. This implies that all senders' reputations would strictly improve at time t, implying that R must not be consistent with Bayes Rule. Contradiction. That $\sigma_t(1,0) = 0$ follows analogously. Next, we show that $\sigma_t(m, \frac{1}{2}) > 0$ for $m \in \{0,1\}$. Suppose by contradiction that $\sigma_t(1, \frac{1}{2}) = 0$. Since we showed above that $\sigma_t(1,0) = 0$, we once again obtain that $R_t(1,1) = R_t(1,0) > R_{t-1}$, implying that all senders would strictly improve their reputations at time t, implying that R_t must violate Bayes' Rule. Contradiction. that $\sigma_t(0, \frac{1}{2})$ follows analogously.

Next, we consider the case where $\lambda_B^t \geq \lambda_G$. Note it follows form the definition of λ_B^t that $\sigma_{t-1}(\emptyset, 1) = \sigma_{t-1}(\emptyset, 0) > 0$. Because the good sender must be acting optimally and good senders truthfully report arrivals, it follows that

$$V_{t-1}(1,1) = V_{t-1}(\emptyset, 1) = V_t(1,1).$$

where the first equality follows from the fact that $\sigma_{t-1}(\emptyset, 1) = \sigma_{t-1}(\emptyset, 0) > 0$. Now suppose by contradiction that $\sigma_t(0,1) > 0$. Then by Lemma 4 $V_t(1,p)$ is weakly increasing in p and $V_t(0,p)$ is strictly decreasing in p. This implies that $\sigma_t(1, \frac{1}{2}) = \sigma_t(1,0) = 0$. Then, again applying Lemma 4, this implies that $V_t(1,p)$ is constant in p. However, by the inductive assumption

$$V_{t-1}(1, \frac{1}{2}) \geq V_{t-1}(\emptyset, \frac{1}{2}) \geq \lambda_B^t V_t(1,1) + (1 - \lambda_B^t) V_t(1, \frac{1}{2}) = V_t(1,1).$$

where the final inequality follows from the fact that $V_t(1,p)$ is constant in p. Combining this with the prior established fact that $V_{t-1}(1,1) = V_t(1,1)$, we obtain that $V_{t-1}(1, \frac{1}{2}) \geq V_{t-1}(1,1)$. However, by the inductive assumption, $\sigma_{t-1}(1, \frac{1}{2}) > 0$, which implies byLemma 4 that $V_{t-1}(1,p)$ is strictly increasing in p. Contradiction. Next, we wish to show that $\sigma_t(m, \frac{1}{2}) > 0$ for $m \in \{0,1\}$. Suppose by contradiction that $\sigma_t(1, \frac{1}{2}) = 0$. By the previously established fact that $\sigma_t(1,0) = 0$, this implies by Lemma 4 that $V_t(1,p)$ is constant in p. We would once again obtain that $V_{t-1}(1, \frac{1}{2}) \geq V_t(1,1)$, implying that $V_{t-1}(1, \frac{1}{2}) \geq V_{t-1}(1,1)$, contradicting the fact that $V_{t-1}(1,p)$ must be strictly increasing in p (by the inductive assumption)

Next, we show that $\sigma_t(\emptyset, 1) = \sigma_t(\emptyset, 0)$. Because we have established that $\sigma_t(0,1) = \sigma_t(1,0) = 0$, it suffices to show that $\sigma_t(1,1) = \sigma_t(0,0)$. Suppose not, i.e., suppose by contradiction that $\sigma_t(1,1) > \sigma_t(0,0)$.

First, I claim that $R_t(0,0) > R_t(1,1)$. Suppose not, i.e., that $R_t(0,0) \leq R_t(1,1)$. By definition of R, this implies that $L_t(1,1) \leq L_t(0,0)$, i.e.,

$$\frac{\lambda_B^t \pi \sigma_t(0,0) + (1-\lambda_B^t)\sigma_t(0,\frac{1}{2})}{\pi \lambda_G} \geq \frac{\lambda_B^t \pi \sigma_t(1,1) + (1-\lambda_B^t)\sigma_t(1,\frac{1}{2})}{\pi \lambda_G}$$
$$\Leftrightarrow \lambda_B^t \pi(\sigma_t(1,1) - \sigma_t(0,0)) \leq (1-\lambda_B^t)(\sigma_t(0,\frac{1}{2}) - \sigma_t(1,\frac{1}{2}))$$
$$\Leftrightarrow \lambda_B^t (1-\pi)(\sigma_t(1,1) - \sigma_t(0,0)) < (1-\lambda_B^t)(\sigma_t(0,\frac{1}{2}) - \sigma_t(1,\frac{1}{2}))$$
$$\Leftrightarrow L_t(0,1) > L_t(1,0)$$
$$\Leftrightarrow R_t(0,1) < R_t(1,0).$$

Thus, it follows that

$$V_t(0,\frac{1}{2}) = \frac{R_t(0,0) + R_t(0,1)}{2} < \frac{R_t(1,1) + R_t(1,0)}{2} = V_t(1,\frac{1}{2}).$$

However, this would violate the above established fact that $\sigma_t(0,\frac{1}{2}) > 0$. Contradiction.

Next I claim that $R_t(1,0) > R_t(0,1)$. To show this, assume by contradiction that that $R_t(0,1) \geq R_t(1,0)$. Given our above result that $R_t(1,1) < R_t(0,0)$, this would imply that

$$V_t(0,\frac{1}{2}) = \frac{R_t(0,0) + R_t(0,1)}{2} > \frac{R_t(1,1) + R_t(1,0)}{2} = V_t(1,\frac{1}{2}).$$

This contradicts the fact that $\sigma_t(1,\frac{1}{2}) > 0$.

Next, we show that $V_t(1,1) < V_t(0,0)$. To see this note that

$$V_t(1,\frac{1}{2}) = V_t(0,\frac{1}{2}) \Leftrightarrow [R_t(1,1) - R_t(0,0)] + [R_t(1,0) - R_t(0,1)] = 0$$
$$\Leftrightarrow \pi[R_t(1,1) - R_t(0,0)] + (1-\pi)[R_t(1,0) - R_t(0,1)] < 0 \Leftrightarrow V_t(1,1) < V_t(0,0)$$

which follows from our earlier observations that $R_t(1,1) < R_t(0,0)$ and $R_t(1,0) > R_t(0,1)$.

Now, note that because by assumption σ_s is silence symmetric for all $s < t$, it follows that the sender's type is uncorrelated the state at time t. Formally, let $R_{t|\theta}$ denote the expected reputation of the sender (from the receiver's perspective), given that at time t she holds prior belief R_{t-1} about the sender's type. Formally,

$$R_{t|\theta} \equiv R_{t-1}[V_t(\theta,\theta)\lambda_G + R_t(1-\lambda_G)] + (1-R_{t-1})[V_t(\theta,\theta)\lambda_B + V_t(\theta,\frac{1}{2})(1-\lambda_B)]$$

If the sender's type is uncorrelated with the state at time t, it follows that $R_{t|\theta=1} = R_{t|\theta=0}$. However, it follows from the expression above that $R_{t|\theta=0} > R_{t|\theta=1}$. This is a contradiction.

Finally, we show that $\sigma_t(1, \frac{1}{2}) = \sigma_t(0, \frac{1}{2})$. Suppose by contradiction that $\sigma_t(1, \frac{1}{2}) > \sigma_t(0, \frac{1}{2})$. Given that $\sigma_t(1, 1) = \sigma_t(0, 0)$, this would imply that both $L_t(1, 1) > L_t(0, 0)$ and $L_t(1, 0) > L_t(0, 1)$, due to the following inequalities:

$$L_t(1,1) = \frac{\sigma_t(1,1)\pi\lambda_B + \sigma_t(1,\frac{1}{2})\frac{1}{2}(1-\lambda_B)}{\pi\lambda_G} > \frac{\sigma_t(0,0)\pi\lambda_B + \sigma_t(0,\frac{1}{2})\frac{1}{2}(1-\lambda_B)}{\pi\lambda_G} = L_t(0,0)$$

$$L_t(1,1) = \frac{\sigma_t(1,1)(1-\pi)\lambda_B + \sigma_t(1,\frac{1}{2})\frac{1}{2}(1-\lambda_B)}{(1-\pi)\lambda_G} > \frac{\sigma_t(0,0)(1-\pi)\lambda_B + \sigma_t(0,\frac{1}{2})\frac{1}{2}(1-\lambda_B)}{(1-\pi)\lambda_G} = L_t(0,0)$$

This in turn implies that, $R_t(1, 1) < R_t(0, 0)$ and $R_t(1, 0) < R_t(0, 1)$, thus

$$V_t(1, \frac{1}{2}) = \frac{R_t(1,1) + R_t(1,0)}{2} < \frac{R_t(0,0) + R_t(0,1)}{2} = V_t(0, \frac{1}{2})$$

Thus, $\sigma_t(1, \frac{1}{2}) = 0$, a violation of the above lemma. □

Lemma 5 (Boundedness of b_t)**.** *If $\sigma_t(\emptyset, 1) = 0$ for all $s \geq t$, then*

$$b_t < \frac{\lambda_G - \lambda_B^t}{1 - \lambda_B^t}$$

Proof. Suppose by contradiction that $b_t \geq \frac{\lambda_G - \lambda_B^t}{1-\lambda_B^t}$.

First consider the case where $t = T$. Then,

$$L_T(\emptyset, 1) = L_T(\emptyset, 0) = \frac{(1-\lambda_B^T)(1-b_T)}{1-\lambda_G} \leq 1$$

where the final inequality follows from our assumption above on b_T. It follows that

$$R_T(\emptyset, 1) = R_T(\emptyset, 0) \geq R_{T-1}.$$

Thus, $V_T(\frac{1}{2}) \geq V_T(\emptyset, \frac{1}{2}) \geq R_{T-1}$. However, we also now that

$$V_T(\emptyset, \frac{1}{2}) \leq V_T(\emptyset, 1) < V_T(1)$$

Where the first inequality follows from part 3 of Lemma 3 combined with Lemma 4. But then it follows from the above inequalities that the bad sender's reputation at the end of period T must on average strictly exceed her reputation at the the beginning of T:

$$\lambda_B^T V_T(1) + (1 - \lambda_B^T) V_T(\frac{1}{2}) > R_{T-1}$$

meaning that the bad sender's reputation on average strictly increases in period T, violating Bayes Rule.

Next, consider the case where $t < T$. Assume by induction that $b_{t+1} < \frac{\lambda_G - \lambda_B^{t+1}}{1-\lambda_B^{t+1}}$. Because by assumption $\sigma_t(\emptyset, 1) = 0$, $\lambda_B^{t+1} = \lambda_B$, and thus the above inequality becomes

$$b_{t+1} < \frac{\lambda_G - \lambda_B}{1 - \lambda_B}.$$

Next, assume by contradiction that $b_t \geq \frac{\lambda_G - \lambda_B^t}{1-\lambda_B^t}$. This immediately implies two things: $R_{t-1} \leq R_t$ and $b_t > b_{t+1}$. I claim this then implies that

$$R_t(1,s) < R_{t+1}(1,s) \text{ for } s \in \{0,1\}. \tag{4}$$

To see why this must hold, first note that

$$R_t(1,1) = \frac{1}{1 + \frac{1-R_{t-1}}{R_{t-1}} \frac{\lambda_B^t \pi + (1-\lambda_B^t) b_t/2}{\lambda_G \pi}}$$

$$R_{t+1}(1,1) = \frac{1}{1 + \frac{1-R_t}{R_t} \frac{\lambda_B^t \pi + (1-\lambda_B^t) b_t/2}{\lambda_G \pi}}$$

where because $R_t \geq R_{t-1}$, $b_t \geq \frac{\lambda_G - \lambda_B}{1-\lambda_B} > b_{t+1}$, and the fact that $\lambda_B^t \geq \lambda_B$, it follows that $R_t(1,1) < R_{t+1}(1,1)$. One can analogously show that $R_t(1,0) < R_{t+1}(1,1)$.

Next, note that

$$V_t(\emptyset, \frac{1}{2}) = qR_{t+1}(1,1) + (1-q)R_{t+1}(1,0)$$

where $q \equiv \pi\lambda_B + \frac{1}{2}(1-\lambda_B) > \frac{1}{2}$. Meanwhile,

$$V_t(1, \frac{1}{2}) = \frac{1}{2}R_{t+1}(1,1) + \frac{1}{2}R_{t+1}(1,0).$$

Then, applying Equation 4 these expressions, we obtain that $V_t(\emptyset, \frac{1}{2}) > V_t(1, \frac{1}{2})$. However, this is a violation of part 2 of Lemma 3. Contradiction. □

Lemma 6 (Truthfully report arrivals)**.** *In any equilibrium, for every t*

$$\sigma_t(1,1) = \sigma_t(0,0) = 1.$$

Proof. Recall that in Lemma 3, we showed $\sigma_t(1,0) = \sigma_t(0,1)$. Thus to prove the proposition, it suffices to show that $\sigma_t(\emptyset, 1) = \sigma_t(\emptyset, 0) = 0$ for all t. By part 1. of Lemma 3, it suffices to show $\sigma_t(\emptyset, 1) = 0$. Suppose by contradiction this is not the case. Let t^* denote the last period such that this condition is not satisfied. Formally,

$$t^* = \max\{t \in \{1, ..., T\} | \sigma_t(\emptyset, 1) = 0\}.$$

First consider the case where $t^* = T$. In this case, because both messages $\emptyset$ and 1 must be optimal under belief p in period T,

$$V_T(\emptyset, 1) = V_T(1, 1)$$

Secondly, by part 3 of Lemma 3:

$$V_T(\emptyset, \frac{1}{2}) \leq V_T(1, \frac{1}{2}).$$

Note further that by Lemma 4 combined with Lemma 3, it follows that $V_T(1, p)$ is strictly increasing in p. Combining this with the two inequalities above, we obtain

$$V_T(\emptyset, 1) > V_T(\emptyset, \frac{1}{2})$$

Since by Lemma 3 it follows that $\sigma_T(\emptyset, 0) > 0$ as well, by analogous reasoning as above, we also obtain that

$$V_T(\emptyset, 0) > V_T(\emptyset, \frac{1}{2}).$$

The above two inequalities imply that $V_T(\emptyset, p)$ is not monotonic in p. However, recall that

$$V_T(\emptyset, p) = p(2\pi - 1)[R_T(\emptyset, 1) - R_T(\emptyset, 0)] + [R_T(\emptyset, 1)(1 - \pi) + R_T(\emptyset, 0)\pi]$$

which is monotonic in p. Contradiction.

Next, consider the case in which $t^* < T$. Because it is optimal for time $t^* + 1$ senders who know the state to report it truthfully,

$$V_{t^*}(1, 1) = V_{t^*}(\emptyset, 1) = V_{t^*+1}(1, 1)$$

where the first equality follows from the fact that $\sigma_{t^*}(\emptyset, 1) > 0$. Furthermore, in order to ensure that faking at time t^* is optimal, which must hold by Lemma 3,

$$\frac{V_{t^*}(1, 1) + V_{t^*}(1, 0)}{2} \geq (\lambda_B + \frac{1 - \lambda_B}{2})V_{t^*+1}(1, 1) + \frac{1 - \lambda_B}{2}V_{t^*+1}(1, 0)$$

This it follows that $V_{t^*}(1, 0) > V_{t^*}(1, 0)$. This combined with our earlier equality $V_{t^*}(1, 1) =$

$V_{t^*+1}(1,1)$ implies that

$$R_{t^*}(1,1) < R_{t^*+1}(1,1) \text{ and } R_{t^*}(1,0) > R_{t^*+1}(1,0)$$

Recalling the above expression for the reputation function above, $R_{t^*}(1,0) > R_{t^*+1}(1,0)$ will hold only if

$$\frac{1-R_{t^*-1}}{R_{t^*-1}}(\sigma_{t^*}(1,1)\lambda_B^{t^*}(1-\pi)+\frac{1}{2}\sigma_{t^*}(1,\frac{1}{2})(1-\lambda_B^{t^*})) < \frac{1-R_{t^*}}{R_{t^*}}(\sigma_{t^*+1}(1,1)\lambda_B^{t^*+1}(1-\pi)+\frac{1}{2}\sigma_{t^*+1}(1,\frac{1}{2})(1-\lambda_B^{t^*+1}))$$

Rearranging, this is equivalent to:

$$[\frac{1-R_{t^*-1}}{R_{t^*-1}}\sigma_{t^*}(1,1)\lambda_B^{t^*} - \frac{1-R_{t^*}}{R_{t^*}}\sigma_{t^*+1}(1,1)\lambda_B^{t^*+1}](1-\pi) <$$
$$\frac{1}{2}[\frac{1-R_{t^*}}{R_{t^*}}\sigma_{t^*}(1,\frac{1}{2})(1-\lambda_B^{t^*+1}) - \frac{1-R_{t^*-1}}{R_{t^*-1}}\sigma_{t^*+1}(1,\frac{1}{2})(1-\lambda_B^{t^*})]$$

Next, I claim

$$\frac{1-R_{t^*-1}}{R_{t^*-1}}\sigma_{t^*}(1,1)\lambda_B^{t^*} - \frac{1-R_{t^*}}{R_{t^*}}\sigma_{t^*+1}(1,1)\lambda_B^{t^*+1} > 0. \tag{5}$$

Suppose by contradiction that the left-hand side is less than or equal to zero. By the above inequality, it would then follow that

$$[\frac{1-R_{t^*-1}}{R_{t^*-1}}(\sigma_{t^*}(1,1)\lambda_B^{t^*} - \frac{1-R_{t^*}}{R_{t^*}}(\sigma_{t^*+1}(1,1)\lambda_B^{t^*+1}]\pi < \frac{1}{2}\sigma_{t^*+1}(1,\frac{1}{2})(1-\lambda_B^{t^*+1}) - \frac{1}{2}\sigma_{t^*}(1,\frac{1}{2})(1-\lambda_B^{t^*})$$

However, this holds if and only if $R_{t^*}(1,1) > R_{t^*+1}(1,1)$, a contradiction of the above.

Now let us examine (5). First, note that since $\sigma_{t^*+1}(1,1) = 1$, and by definition of λ_B^t, it must be that $\lambda_B^{t^*}\sigma_{t^*}(\emptyset,1) < \lambda_B^{t^*+1}$, it must be that $R_{t^*-1} < R_{t^*}$. From our earlier expression for R_t, this implies an upper bound on $\sigma_{t^*}(1,\frac{1}{2})$

$$1-\lambda_G > (1-\lambda_B^{t^*})(1-\sigma_{t^*}(1,\frac{1}{2})) + \lambda_B^{t^*}\sigma_{t^*}(\emptyset,1) \Leftrightarrow \sigma_{t^*}(1,\frac{1}{2}) > \frac{1-\lambda_B^{t^*+1}}{1-\lambda_B^{t^*}}\sigma_{t^*+1}(1,\frac{1}{2}) \tag{6}$$

Separately, recalling (5), the only way to ensure that both $R_{t^*}(1,1) < R_{t^*+1}(1,1)$ and $R_{t^*}(1,0) > R_{t^*+1}(0,1)$ is if

$$\sigma_{t^*}(1,\frac{1}{2}) < \frac{1-\lambda_B^{t^*+1}\sigma_{t+1}(1,\frac{1}{2})}{1-\lambda_B^{t^*}}. \tag{7}$$

Next, recall by Lemma 5 that $b_{t^*+1} < \frac{\lambda_G - \lambda_B^{t^*+1}}{1-\lambda_B^{t^*+1}}$ substituting this into (7), we have

$$\sigma_{t^*}(1, \frac{1}{2}) < \frac{\lambda_G - \lambda_B^t \sigma_t(\emptyset, 1) - (1 - \lambda_B^t \sigma_t(\emptyset, 1))\lambda_B}{1 - \lambda_B^t},$$

which contradicts (6). □

Proof of Proposition 4. First, we show that in equilibrium $\sigma_t(0,0) = \sigma_t(1,1) = 1$. This follows directly from part 2 of Lemma 3 and Lemma 6. Next, we must show that $b_t \equiv \sigma_t(1, \frac{1}{2}) = \sigma_t(0, \frac{1}{2}) > 0$ at all t, which is given directly by Lemma 3. It remains to show that $b_t < 1$. Suppose not by contradiction. Then it follows that the sender can guarantee a reputation of 1 by abstaining at t, regardless of her information. This is due to the fact that $b_t = 1$ implies that $R_t = 1$, and thus it follows from Bayes' Rule that $R_\tau(m, s) = 1$ for all m, s, and $\tau \geq t$. Thus, abstaining at t serves as a profitable deviation. Contradiction.

Now, I establish existence of an equilibrium. I do so by invoking the Kakutani fixed point theorem. As per the first part of the proof, we can restrict attention to strategies of the form σ^b for the bad sender, where $b \in [0,1]^T$. Per the selection assumption, suppose that for all t $\sigma_t^G(1,1) = \sigma_t^G(0,0) = \sigma_t^G(\emptyset, \frac{1}{2}) = 1$. I will later confirm that in the candidate equilibria I identify, this strategy is optimal for G. I begin by defining a best response correspondence. To this end, let R^b denot the unique reputation function that is consistent with B playing σ^b, and G playing the truth-telling strategy above. Let $V_t^{i,b}$ denote the value function for player i given reputation function R^b. Let $\psi_t : [0,1]^T \to 2^{[0,1]}$ denote the best response correspondence for an uninformed B sender at time t given reputation function R^b. Let $\varphi \equiv \psi_1 \times ... \times \psi_T$. I claim that φ has a fixed point. To this end, note that

$$\psi_t(b) = \begin{cases} 1 & \text{if } V_t^{B,b}(1, \frac{1}{2}) > V_t^{B,b}(\emptyset, \frac{1}{2}) \\ 0 & \text{if } V_t^{B,b}(1, \frac{1}{2}) < V_t^{B,b}(\emptyset, \frac{1}{2}) \\ [0,1] & \text{if } V_t^{B,b}(1, \frac{1}{2}) = V_t^{B,b}(\emptyset, \frac{1}{2}). \end{cases} \tag{8}$$

Thus, $\psi_t(b)$ is convex and non-empty for all b. it follows that $\psi(b)$ is also convex and non-empty for all b. By the same reasoning as in the static case, $V_t^{B,b}$ is continuous in b for all $m \in \{1, \emptyset\}$. It thus follows from (8) that ψ is upper hemi-continuous in b. It thus follows from the Kakutani fixed point theorem that φ has a fixed point. I now claim that any fixed point must lie in $(0,1)^T$. This follows directly from the first part of the proof. Take any such fixed point $b \in (0,1)^T$. I claim that σ^G, σ^b, R^b is an equilibrium, thus establishing existence. To this end, let us begin by showing tht the bad sender cannot profitably deviate. Fix a t and consider a bad sender who has not yet learned the state, and thus holds belief $\frac{1}{2}$.

It follows from the fact that b is a fixed point of ψ that $V_t^{B,b}(1, \frac{1}{2}) = V_t^{B,b}(\emptyset, \frac{1}{2})$ and thus $\sigma_t^B(\emptyset, \frac{1}{2}) = b/2$ is a best response. Next, consider a bad sender who has learned the state is 1, and thus holds belief $p = 1$. We want to show

$$V_t^{B,b}(1,1) > V_t^{B,b}(\emptyset, 1) \qquad\qquad V_t^{B,b}(1,1) > V_t^{B,b}(0,1)$$

The second statement holds because, given the strategy profile of the senders, $R_t^b(1,1) > R_t^b(0,1)$. The first statement holds because, given the strategy profile of the senders, $R_t^b(1,1) > R_{t+1}^b(1,1)$ for all $t < T$. It remains to show that the good sender cannot profitably deviate. The good, informed sender cannot profitably deviate for the same reason that the bad informed sender cannot deviate (they face the same value functions). Now, let us consider the uninformed, good sender. Note that

$$V_t^{G,b}(1, \frac{1}{2}) = V_t^{B,b}(1, \frac{1}{2}) = V_t^{B,b}(\emptyset, \frac{1}{2}) \leq V_t^{G,b}(\emptyset, \frac{1}{2}).$$

Thus, the good sender cannot profitably deviate at belief $\frac{1}{2}$. □

Proof of Proposition 6. Combining Lemma 6 and Lemma 5, it follows that for all t,

$$b_t < \frac{\lambda_G - \lambda_B}{1 - \lambda_B}.$$

Recall further that it follows from Lemma 3 that

$$R_t = \frac{1}{1 + \frac{1-R_{t-1}}{R_{t-1}} \frac{(1-\lambda_B)(1-b_t)}{1-\lambda_G}}$$

Combining this equality with the previous inequality on b_t implies the statement. □

Proof of Proposition 7. Suppose by contradiction that $b_t \leq b_{t+1}$. Under the equilibrium strategy, for any $\tau \in 1, ..., T$ and $s \in \{0, 1\}$,

$$L_\tau(1,1) = \frac{Pr(s|\theta = 1)\lambda_B + (1 - \lambda_B)b_\tau/2}{\lambda_G Pr(s|\theta = 1)}$$

It follows from the assumption that $L_t(1, s) \leq L_{t+1}(1, s)$ for all s. Recalling that

$$R_t(1, s) = \frac{1}{1 + \frac{1-R_{t-1}}{R_{t-1}} L_t(1, s)}$$

Proposition 6 above then implies that $R_t(1, s) < R_{t+1}(1, s)$ for $s \in \{0, 1\}$.

Next, recall the value of the uninformed sender at time t from reporting 1 is given by:

$$V_t(1, \frac{1}{2}) = \frac{R_t(1,1) + R_t(1,0)}{2}$$

Meanwhile, her value from abstaining at t is given by:

$$V_t(\emptyset, \frac{1}{2}) = (\lambda_B \pi + (1 - \frac{\lambda_B}{2})R_{t+1}(1,1) + \frac{1-\lambda_B}{2}R_{t+1}(1,0)$$

It follows from the above inequalities that there exists a $\overline{\lambda_B}$ such that if $\lambda_B < \overline{\lambda_B}$, $V_t(\emptyset, \frac{1}{2}) < V_t(1, \frac{1}{2})$. However, this is a contradiction of the equilibrium characterization above, which requires indifference between messages $\emptyset$ and 1 when $p = \frac{1}{2}$.

□

Proof of Proposition 5. First, we establish that for all t, $\alpha_t > 0$. Suppose not, by contradiction. Let us begin by establishing a set of inequalities. First, note that

$$R_t(1,1) = R_{t-1} + \alpha_t \leq R_{t-1}.$$

Separately, $R_t(1,1) > R_t(1,0)$. This holds by identical reasoning to what is presented in Lemma 2 (i.e., the static case). This implies that $R_t(1,0) < R_{t-1}$. Finally, by Proposition 6, $R_t < R_{t-1}$.

Now, assume that at time $t-1$, the sender has not yet reported. Then, the receiver's expected time-t belief about the sender's type at time $t-1$ is given by

$$\begin{aligned} E_{t-1}[Pr_t[\theta = G]] = 2Pr_{t-1}(m_t = 1 = s)R_t(1,1) + 2Pr_{t-1}(m_t = 1 \neq s)R_t(1,0) \\ +(1 - 2Pr_{t-1}(m_t = 1 = s) - 2Pr(m_t = 1 \neq s))R_t \end{aligned}$$

Since $\lambda_B < 1$ and $b_t < 1$ (by Proposition 4), $Pr_{t-1}(m_t = 1 \neq s) > 0$. Combining this with the above-established inequalities that $R_t(1,1) \leq R_{t-1}$, $R_t(1,0), R_{t-1}$, and $R_t < R_{t-1}$ yields

$$E_{t-1}[Pr_t[\theta = G]] < R_{t-1} = Pr_{t-1}[\theta = G].$$

This is the violation of the martingale property of the receiver's belief about the sender's type. Contradiction.

Now, we claim that for all t, $\alpha_t < -\beta_t$. Note that having established that $\alpha_t > 0$, this also implies that $\beta < 0$. To show this, suppose by contradiction that $\alpha_t \geq -\beta_t$. Then,

$$V_t^i(1) = V_t^i(1,1) = R_{t-1} + \pi\alpha_t - (1-\pi)\beta_t > R_{t-1} \text{ for all } i \in \{B, G\},$$

where the first equality follows from Proposition 4 and the strict inequality follows from the above-established fact that $\alpha_t > 0$. Separately,

$$V_t^B(\frac{1}{2}) = V_t^B(\frac{1}{2}, 1) = R_{t-1} + \frac{1}{2}\alpha_t - \frac{1}{2}\beta_t \geq R_{t-1}$$

Finally, I claim that $V_t^G(\frac{1}{2}) \geq V_t^B(\frac{1}{2})$. We prove this using backwards induction. Note that in the base, case we have

$$V_T^G(\frac{1}{2}) = V_T^B(\frac{1}{2}) = R_T(\emptyset)$$

where the final inequality again follows from Proposition 4. Now, fix any $t < T$, and suppose by induction that $V_{t+1}^G(\frac{1}{2}) \geq V_{t+1}^B(\frac{1}{2})$. We want to show that $V_t^G(\frac{1}{2}) \geq V_t^B(\frac{1}{2})$. To this end, note that

$$V_t^i(\frac{1}{2}) = V_t^i(\frac{1}{2}, \emptyset) = \lambda_i V_{t+1}^i(1,1) + (1-\lambda_i)V_{t+1}^i(\frac{1}{2})$$

Then

$$V_t^G(\frac{1}{2}) - V_t^B(\frac{1}{2}) = (\lambda_G - \lambda_B)[V_{t+1}^G(1,1) - V_{t+1}^B(1,1)] + (1-\lambda_G)[V_{t+1}^G(\frac{1}{2}) - V_{t+1}^B(\frac{1}{2})]$$

Since $V_{t+1}^G(\frac{1}{2}) \geq V_{t+1}^B(\frac{1}{2})$ by the inductive assumption, it suffice to show $V_{t+1}^G(1,1) \geq V_{t+1}^B(\frac{1}{2})$. This indeed holds, since

$$V_{t+1}^B(\frac{1}{2}) = V_{t+1}^B(\frac{1}{2}, 1) = \frac{1}{2}R_{t+1}(1,1) + \frac{1}{2}R_{t+1}(1,0) < \pi R_{t+1}(1,1) + (1-\pi)R_{t+1}(1,0) = V_{t+1}^G(1,1).$$

Now, once again assume that at time t, the sender has not yet reported. Then it follows from Bayes Rule that

$$\begin{aligned} E_{t-1}[Pr_T[\theta = G]] &= Pr_{t-1}[\theta = G]E_{t-1}[Pr_T[\theta = G]|\theta = G] + Pr_{t-1}[\theta = B]E_{t-1}[Pr_T[\theta = G]|\theta = B] \\ &= R_{t-1}(\lambda_G V_t(1,1) + (1-\lambda_G)V_t^G(\frac{1}{2})) + (1-R_{t-1})(\lambda_B V_t(1,1) + (1-\lambda_B)V_t^B(\frac{1}{2})) \end{aligned}$$

Since $\lambda_G > 0$ and $\lambda_B > 0$, it follow sfrom the above inequalities that $E_{t-1}[Pr_T[\theta = G]] > R_{t-1} = Pr_{t-1}[\theta = G]$. This is a violation of the Martingale property of the receiver's belief about the sender's type. Contradiction.

□