Homophily in Social Media and News Polarization*

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Abstract

We consider an ad-financed media firm that chooses the ideological location of its news when consumers who directly receive the news can share it with their followers on social media. When the firm maximizes the breadth of news sharing, it tends to produce polarized news if the mean (the variance) of ideological locations of the followers of a direct consumer is a convex (concave) function of the latter's location. This implies that it is the curvature, rather than the slope, of homophily that determines news polarization so that surprisingly, larger homophily at the center (extremes) can lead to (no) polarization.

Keywords: Media Bias, Social Media, Homophily, News Sharing, Polarization

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"What information consumes is rather obvious: it consumes the attention of its recipients. Hence a wealth of information creates a poverty of attention..."

(Simon, 1971)

1 Introduction

Social media has become a very important source for news consumption. According to Liedke and Wang (2023), as of October 2023, two-thirds of Americans report that they consume news on social media at least sometimes– with two in ten doing so often. Among different social media, Facebook remains the most relevant: 30% of Americans regularly consume news on Facebook.

This success of social media, however, is viewed with suspicion as some fear that the tendency of social media users to mainly consume like-minded news would prevent them from digesting diverse viewpoints about important issues. Although people express such concerns by pointing out problems such as filter bubbles (Pariser, 2011) or echo chambers (Sunstein, 2017), one might think that homophily or social network structure is a more fundamental source of the problem. Indeed, by analyzing data from US Facebook users, Bakshy, Messing and Adamic (2015) found that homophily is the most important factor limiting their exposure to attitude-challenging content.[1] Even if there is a consensus that homophily on social media leads to more consumption of like-minded news, there is mixed evidence about whether social media leads to political polarization; Barberá (2015) and Boxell, Gentzkow and Shapiro (2017) find evidence inconsistent with polarization, while Allcott et al. (2020), Levi (2021), and Yanagizawa-Drott, Petrova and Enikolopov (2019) find evidence consistent with polarization.

This paper contributes to the debate on the relationship between social media and news polarization by focusing on the supply of news and addresses the following questions. How does the structure of social networks on social media affect news sharing and thereby shape the incentive for a media firm to choose the ideological bias of its news? Under what conditions is the media firm incentivized to provide polarized news? Indeed, Kümpel, Karnowski

[1]Similarly, Halberstam and Knight (2016) analyzed information from 2.2 million Twitter users on the day before the 2012 US general elections and found that, due to homophily, people are disproportionately exposed to tweets from like-minded others.

and Keyling (2015), who review 109 papers on news sharing on social media published by communication or computer scientists, call for a theory emphasizing the role of the social networks of followers on the decision to share news. This is exactly what we attempt to accomplish.

To answer the above questions, we consider a three-tiered hierarchy: a media firm, direct consumers and their followers on social media. The media firm sends its news to targeted direct consumers, who in turn decide whether to share the news with their followers. We show that the incentive for a direct consumer to share a given news item depends on the ideological location of the news as well as the mean and the variance of the locations of her followers, which in turn depend on the direct consumer's own ideological position. Therefore, to understand news polarization, it is important to know how extremists' followers differ from those of moderates in terms of the mean and the variance of their locations. In fact, it is known that ideologically more extreme individuals tend to be more homophilous than more moderate ones in the sense of lower variance (Boutyline and Willer, 2017). As our main result, we find that when the mean (respectively, the variance) of followers' ideological locations is a convex (respectively, concave) function of a direct consumer's location, the media firm is likely to produce polarized news. In contrast, when the mean is concave or the variance is convex, it is likely to produce unbiased news.

Our questions are motivated by anecdotal evidence from the US presidential election in 2016. Over one hundred websites with false content were created by teenagers from Macedonia, seeking advertising revenues propelled by sharing of their news on Facebook (Silverman and Alexander, 2016). The teenagers were using Facebook to drive traffic to their websites where they had ads from Google. Their sites produced misleading partisan content and obtained more engagement than op-eds and commentary pieces from major media (NPR, 2016). Even though our model does not distinguish between fake news and true news, our results help to understand when polarized news is more likely to be shared than less polarized news.

In Section 2, we present our baseline model, in which consumers are distributed over an interval $[0, 1]$ of ideological space and a media firm chooses the ideological location of its news to maximize its advertising revenue, which is proportionate to the measure of readers. We assume that the utility from reading the news has a quadratic loss. Direct consumers receive news from the media firm and are distributed over $[0, 1]$ according to a distribution function, which is symmetric around 1/2. We assume that a direct consumer shares the news if it is relevant enough to their followers, in that the average benefit across their follower base exceeds the attention tax she imposes on them by sharing the news. The *attention tax* captures the opportunity cost of attention, as sharing news with a friend induces the latter to pay attention to it. This sharing behavior can be microfounded by a setting where sharers care about their social status and thus receive utility from positive reactions (e.g. "likes") to their shared content.

In Section 3, for simplicity, we assume that the constant utility from reading the news is large enough that all consumers who have access to the news read it. Under this assumption, profit maximization of the media firm is equivalent to maximizing the measure of the direct consumers who share the news, which we define as the *breadth of news sharing.* If the attention tax is low enough, by locating the news at the middle, the media firm can induce all direct consumers to share the news. Hence, we focus on the case in which the attention tax is not small. As an important intermediary result, we show that a direct consumer's benefit from news sharing decreases with the ideological dispersion of her followers and with the mismatch between the location of the news and the mean location of her followers. As a main result, we find that news polarization is likely to occur if the mean of the followers' ideological locations is a convex function of a direct consumer's location and/or the variance is a concave function. To provide an intuition for why a convex mean leads to news polarization, suppose that the mean is increasing and convex over $[0, 1/2]$.[2] This implies that the mean followers of two left-wing extremists are more closely located than those of two moderates. This can induce news polarization because, as the interval of direct consumers who share a news item expands, the marginal consumer's benefit from sharing it decreases more slowly when the news is located close to the extremists than when it is located close to the moderates. However, polarization does not entail choosing a news location equal to 0 or 1 because the media firm does not need further polarization of the news once it is extreme enough to induce the most extreme direct consumer to be indifferent between sharing the news and not, which we call the *limit polarization.*

When we interpret the above-mentioned main result in terms of a measure of homophily which we introduce, surprisingly, we find that when homophily increases (decreases) as consumers become less extreme, this can lead to limit polarization (no polarization). More

[2] We consider a symmetric distribution of both direct readers and their followers around 1/2.

generally, we show that it is the curvature, rather than the slope, of homophily that is relevant for polarization: when homophily is sufficiently convex (concave), there is polarization (no polarization).

In Section 4, we study competition between two media firms: firm L and firm R. All direct consumers receive a news item from each firm, but each consumer can share at most one news item. We find that under mild conditions, as long as polarization occurs in the baseline model without competition, polarization also occurs under competition. When firm R's news becomes more polarized, additional right-wing consumers share its news whereas some moderate consumers stop sharing it. Compared to the same change in news location in the baseline model without competition, we find that the gain of right-wing consumers is the same whereas the loss of moderate consumers is typically smaller under competition. This is because the benefit that a moderate obtains by exercising the option of sharing firm L's news decreases as her position moves to the right whereas in the baseline model, consumers have no outside option, which is akin to having an outside option that does not depend on their location.

In Section 5, we provide three extensions of the baseline model. The first allows the firm to target an interval of consumers to show the news. When direct consumers are distributed uniformly, we find that it is a weakly dominant strategy for the firm to choose the same location of news as in the baseline model and to make the left end of the target interval coincide with the left marginal consumer who is indifferent between sharing the news and not. In the second extension, we add one additional layer of followers to consider resharing the shared news. We define a measure of the connectedness between the first-layer followers and their followers and show that news polarization in the baseline model implies news polarization in the extension as long as this measure of connectedness increases as consumers become more extreme. In the third extension, we study the depth-maximization strategy. We allow for many layers of followers and characterize the strategy that maximizes the depth of sharing, i.e., the number of times the news is shared following down the layers of communication. We find that as long as the attention tax is not small, depth maximization requires targeting the direct consumer with the lowest variance and locating the news at the mean location of the targeted consumer's followers.

Our work is closely related to the literature on demand-driven media bias (Gentzkow, Shapiro and Stone, 2015), in which biased news results from biased consumer beliefs. We contribute more specifically to the models of psychological utility, in which consumers enjoy reading news that confirms their beliefs. As in this paper, both Mullainathan and Shleifer (2005) and Gabszewicz, Laussel and Sonnac (2001) assume psychological utility in the context of a Hotelling-type model to study media bias. In Mullainathan and Shleifer (2005), firms engage in price competition, and bias consists of newspapers slanting their news away from the the truth. They find that under competition and heterogeneous consumer beliefs, firms adopt a maximal differentiation strategy. Meanwhile, in Gabszewicz, Laussel and Sonnac (2001), firms earn a portion of their revenue from advertising, which mitigates differentiation, inducing firms to take similar centrist stances. To the best of our knowledge, we are the first to embed social networks of followers into a model of demand-driven media bias to study how news sharing and news bias are shaped by the characteristics of the social networks.

Empirical papers on demand-driven media bias include Gentzkow and Shapiro (2010) and Larcinese, Puglisi and Snyder (2011). Gentzkow and Shapiro (2010) find that readers have an economically significant preference for like-minded news and that firms respond strongly to consumer preferences. Larcinese, Puglisi and Snyder (2011) find evidence that newspapers cater to readers' partisan tastes on news about unemployment, trade deficits and budget deficits.

The literature on social media is mainly empirical and studies how social media affects voting, protests, xenophobia, polarization and the consumption of fake news (see Zhuravskaya, Petrova and Enikolopov, 2020, for a survey). Our paper is more related to the empirical papers studying polarization, which we reviewed at the beginning of the introduction.[3] In particular, a study by Levi (2021) based on a large field experiment on Facebook finds that Facebook's newsfeed algorithm limits exposure to counter-attitudinal news and thus increases polarization. There are several theoretical papers on social media. In terms of the categories of players, our paper is similar to Fainmesser and Galeotti (2021), which studies the interaction between marketers, influencers and followers. We instead study the interaction between media firms, direct consumers and followers, where direct consumers in our model play the same role as influencers in their model. Despite this common point, the two papers have entirely different objectives: whereas we are interested in news polarization,

[3] See Tucker et al. (2018) for a review of the literature on social media, political polarization and political disinformation.

they instead study influencers' choice between sponsored and organic content. Berman and Katona (2020) study how curation algorithms of social media affect polarization when each receiver chooses the number of senders to follow and each sender chooses the quality of her content. Contrary to the empirical finding of Levi (2021), they find that curation algorithms can reduce polarization as they induce readers to follow a larger number of senders. De Cornière and Sarvary (2023) study how content bundling by social media, i.e., social media showing news content together with user-generated content (UGC), affects the profit of newspapers and their incentive to invest in quality. However, they consider neither news sharing nor the network structure of followers. Meanwhile, like our paper, Acemoglu, Ozdaglar and Siderius (2023) and Kranton and McAdams (2024) study the link between social networks and news sharing. In Acemoglu, Ozdaglar and Siderius (2023), a story of unknown veracity is seeded in a network and agents with different ideological biases decide whether to share it. They characterize the engagement-maximizing network structure, finding that homophilous networks are more common under higher polarization. While they take the news as given and study the strategic choice of network by social media platforms, we take the network structure as given and study the strategic choice of news by news producers. Such is also studied by Kranton and McAdams (2024), who consider the effect of social connectedness on news veracity when news producers incur a cost to producing true news.[4] They find that veracity is highest when social connectedness is neither too high nor too low. Like our paper, they also model producer revenue as being proportional to the number of readers, which is determined by the number of consumers who choose to share the news. Relatedly, Cisternas and Vásquez (2022) study the dissemination of fake news on a social media platform, where the producers of fake news seek to maximize viewership and consumers decide whether to both verify and share the news. They find that reducing verification costs may have unintended consequences by inducing individuals who would otherwise ignore fake news to start

[4]There are other papers studying news sharing in social networks. Bloch, Demange and Kranton (2018) study the transmission of rumors on social networks in a model with two possible states of nature and two types of agents (unbiased or biased). They find that a social network can serve as a filter: unbiased agents block messages from parts of the network that contain many biased agents. Campbell, Leister and Zenou (2024) build a dynamic model with two types of news (mass market and niche market) and two types of individuals. Each individual receives news from randomly sampled friends and shares one news item. They find that greater connectivity and homophily concurrently increase the prevalence of the niche market content and polarization.

sharing it. What distinguishes us from these papers, and the rest of the literature, is that we consider the effect of network structure on polarization. Furthermore, we model a rich structure of followers characterized by the mean and variance of their ideological distributions to study how characteristics of these distributions affect news polarization.

The next section presents our baseline model. All the proofs, but for those of Lemma 1 and Proposition 7 which are presented in the main text, are gathered in Appendix.

2 The Baseline Model

In this section, we present our baseline model, which has three categories of players: a media firm, direct consumers and followers.

The media firm and consumers The media firm chooses the ideological location of its news $y \in [0, 1]$. We consider free news financed by advertising revenue. There is a continuum of consumers, who are also users of social media. Each consumer is located on the interval $[0, 1]$. The utility that a consumer located at x obtains from reading a news located at y is

$$U(x, y) = u - (x - y)^2,$$

where $u > 0$ and $(x-y)^2$ captures the disutility from the mismatch between the news opinion and the consumer's ideal opinion. Consumers read the news whenever they have access to it and obtain a nonnegative utility from reading it.

Direct consumers and followers Each direct consumer receives news from the media firm. Direct consumers are distributed over $[0, 1]$ according to the distribution function F with density $f > 0$, which is assumed to be symmetric around $1/2$.

Each direct consumer has followers, who are also called indirect consumers. Those who follow a direct consumer have access to the news only if it is *shared* by the direct consumer. We assume that every direct consumer has a *distinct* group of followers of equal measure, which is normalized to one. Given a direct consumer located at x, her followers are distributed according to density $\tilde{g}(\,\cdot\,; x)$. We assume that the distribution of followers $z \in [0, 1]$ of a consumer located at x is symmetric to the distribution of followers of a consumer located at $1 - x$ such that $\tilde{g}(z; x) = \tilde{g}(1 - z; 1 - x)$.

News sharing We assume a direct consumer shares a news item if she reads it *and* if the average benefit that her followers obtain from reading it is larger than the attention tax τ she imposes on them by sharing the news. The attention tax represents the opportunity cost of attention, as news sharing by a direct consumer induces each follower to pay attention to it, which does not necessarily mean reading the whole news article. Using Simon (1971)'s expression, we assume that sharing news "consumes the attention of its recipients". A follower can discover the values of u and y by paying attention to the news and then decide whether to read it. Formally, let $B(x, y)$ denote the average benefit that the followers of a direct consumer located at x obtain when the latter shares news located at y. We have

$$B(x, y) = \int_0^1 \max\{U(z, y), 0\} \; \tilde{g}(z; x) \, dz. \tag{1}$$

We thus assume that a direct consumer shares the news whenever $B(x, y) \geq \tau$.

The existing literature from communication science and computer science identifies two motives for news sharing: an altruistic motive (Boyd, Golder and Lotan, 2010; Small, 2011; Holton et al., 2014) and a self-serving motive (Boyd, Golder and Lotan, 2010; Ma, Lee and Goh, 2011; Lee and Ma, 2012). The sharing rule we specify is consistent with both class of motives. Specifically, this sharing rule can be reconciled with standard utility maximization based on self-serving motives such as status seeking where the sharer attaches positive (negative) utilities to the positive (negative) reactions from her followers such as *likes* (*dislikes*) on Facebook or *hearts* on Twitter. [5]

Profit The media firm maximizes its advertising revenue, which is proportional to the traffic to the firm's news site (Gentzkow, Shapiro and Stone, 2015). The traffic is equivalent

[5]Suppose that a direct consumer receives a *like* (resp., *dislike*) from a follower located at z whenever $U(z, y) \geq \tau$ (resp., $U(z, y) < \tau$) and that the difference $U(z, y) - \tau$ (resp., $\tau - \max\{U(z, y), 0\}$) measures the intensity of the feedback received. Then, we have:

$$\mathbb{E}(\text{intensity of likes}) = \int_{\substack{U(z,y)\geq\tau \\ z\in[0,1]}} (U(z, y) - \tau) \; \tilde{g}(z; x) \, dz,$$

and

$$\mathbb{E}(\text{intensity of dislikes}) = \int_{\substack{U(z,y)<\tau \\ z\in[0,1]}} (\tau - \max\{U(z, y), 0\}) \; \tilde{g}(z; x) \, dz.$$

If a direct consumer shares news if and only if $\mathbb{E}(\text{intensity of likes}) - \mathbb{E}(\text{intensity of dislikes}) \geq 0$, then she shares news if and only if $B(x, y) - \tau \geq 0$.

to the measure of readers. Given the location of its news y, the profit of the firm is given by

$$\pi(y) = (1-\alpha)\, D_0(y) + \alpha\, D_1(y),$$

where $D_0(y)$ and $D_1(y)$ are the demand from the direct readers and that from the indirect readers, respectively, and $\alpha \in (0,1)$ captures the importance the firm assigns to the indirect demand relative to the direct one.

We assume that u is large enough that every consumer has an incentive to read the news.

Assumption 1. *u is large enough that $U(x,y) > 0$ for any $(x,y) \in [0,1]^2$.*

This assumption greatly simplifies our analysis because under the assumption, every direct consumer reads the news, and every follower also reads it as long as the news is shared. Therefore, profit maximization is equivalent to maximizing the measure of direct consumers who share the news, which is called the breadth of news sharing:

Definition 1 (Breadth of News Sharing)**.** *The breadth of sharing a news item is the measure of direct consumers who share it.*

In the next section, we study the news location which maximizes the breadth of sharing. Due to the symmetry of the problem, if y^* maximizes total demand, so does $1-y^*$. Therefore, it is sufficient to conduct analysis only over half of the interval. Henceforth, we restrict attention to $y \in [0, 1/2]$ without loss of generality.

3 Breadth-maximizing location: No targeting

In this section, we study the breadth-maximizing location of news in the case of no targeting, which means that the target interval is equal to $[0,1]$. Targeting is considered in Section 5.1. After analyzing the baseline model, we interpret our findings in terms of homophily in Section 3.2.

3.1 Analysis of the baseline model

Let $\mu(x)$ and $\sigma^2(x)$ represent the mean and the variance, respectively, of the locations of the followers of a direct consumer located at x. Precisely,

$$\mu(x) = \int_0^1 z\ \tilde{g}(z;x)\, dz \qquad \text{and} \qquad \sigma^2(x) = \int_0^1 (z-\mu(x))^2\ \tilde{g}(z;x)\, dz.$$

We consider a general model in which the main primitives are the distribution of $\mu(x)$ and $\sigma^2(x)$ over $x \in [0,1]$. The next lemma shows that the density of followers $\tilde{g}(z;x)$ affects $B(x,y)$ only via its mean $\mu(x)$ and variance $\sigma^2(x)$. This is due to a quadratic loss in the utility from reading a news item.

Lemma 1. *Under Assumption 1, the average benefit that the followers of a direct consumer located at x obtain when the latter shares a news item located at y is $B(x,y) = u - \sigma^2(x) - (y - \mu(x))^2$.*

Proof.

$$\begin{aligned} B(x,y) &= \mathbb{E}_z \left(u - (z-y)^2\right) \\ &= u - \mathbb{E}_z \left((z - \mu(x)) - (y - \mu(x))\right)^2 \\ &= u - \sigma^2(x) - (y - \mu(x))^2 \end{aligned}$$

□

Therefore, given a direct consumer located at x, the benefit of news sharing decreases with the ideological dispersion of her followers $\sigma^2(x)$ and with the mismatch between the location of the news and the mean location of her followers $(y - \mu(x))^2$. This result is intuitive. Even if there is little to no mismatch between the news and the mean follower, high dispersion between followers implies that the average mismatch will be high.

We below introduce a technical assumption that adds regularity to $B(\cdot, y)$, thus simplifying our analysis. Under the assumption, the set of locations of direct consumers that share a news item is an interval, that is, if two direct consumers of distinct ideological locations are willing to share a news item, then any direct consumer located between the two is also willing to do so. We also assume that small changes in the ideological slant of a news item induce small changes in direct consumers' willingness to share it.

Assumption 2. *For any y, $B(\cdot, y)$ is strictly quasiconcave and differentiable.*

The quasi-concavity assumption on $B(\cdot, y)$ is not just technical, and has some economic implications: if a moderate is not willing to share a news item, then one of the two extremists definitely will not be willing to share it. In other words, it loosely places some "ordering" across direct consumers in their benefit from sharing a given item. It follows from the expression for $B(x,y)$ in Lemma 1 that assuming homophily (in the sense that $\mu(x)$ is increasing in x), with a constant variance, implies the quasi-concavity. Let $x_l(y)$ and $x_r(y)$, with $x_l(y) \leq x_r(y)$, denote the limits of the interval of direct consumers who share a news item located at $y \in [0, 1]$. From Assumption 1, every follower reads the news if she has access to it. Thus, the indirect demand for a news item located at y is equal to its breadth of sharing, that is,

$$D_1(y) = \int_{x_l(y)}^{x_r(y)} f(x)\, dx = F\left(x_r(y)\right) - F\left(x_l(y)\right).$$

Let y^* represent the breadth-maximizing news location. We first identify a straightforward case of no polarization.

Proposition 1. *Under Assumptions 1 and 2, if the attention tax is low enough that even the most extreme consumer is willing to share news located at* $1/2$*, then no polarization* $y^* = 1/2$ *maximizes the breadth of sharing because* $D_1(1/2) = 1$*.*

Thus, we henceforth consider the more interesting case where no polarization does not ensure full breadth of news sharing. Namely, we assume that the attention tax is high enough to preclude this. This is formalized as Assumption 3.

Assumption 3. $\tau > B(x = 0, y = 1/2)$.

In this case, it is always possible to have a location y (around $1/2$) such that both $x_l(y) > 0$ and $x_r(y) < 1$. The locations where either $x_l(y) = 0$ or $x_r(y) = 1$ are of special interest and will be called the limit polarization locations.

Definition 2 (Limit polarization locations)**.** *Let* $\underline{y}$ *(resp.,* $\overline{y}$*) denote the closest location from the center such that* $x_l(\underline{y}) = 0$ *(resp.,* $x_r(\overline{y}) = 1$*). We will refer to* $\underline{y}$ *as the left limit-polarization location and to* $\overline{y}$ *as the right limit-polarization location.*

From the symmetry of the distribution of followers, $\underline{y} = 1 - \overline{y}$.

We introduce our last assumption.

Assumption 4. *For all* x*:*

1. $\mu(x)$ *is strictly increasing in* x*;*

2. $\sigma^2(x) < u - \tau$.

Assumption 4 imposes two conditions on the distributions of followers which each capture different notions of homophily. First, $\mu(x)$ being strictly increasing ensures that direct consumers are similar to their followers on average. More precisely, if one direct consumer is more right-leaning than another, her followers will be more right-leaning on average as well. The second condition places an upper bound on $\sigma^2(x)$, and thus ensures that for any given direct consumer, her followers will be sufficiently similar to each other. These two conditions are relevant to the analysis because they ensure that if followers are sufficiently close to the location of news y on average, the direct consumer will share the news with them. If this were not the case, then moving away from y would ensure an increase in breadth. Therefore, the assumption makes breadth maximization less trivial.

Assumption 4 implies the following intuitive result:

Lemma 2. *If $y \geq \underline{y}$ then $y > \mu(x_l(y))$. If $y \leq \overline{y}$, then $y < \mu(x_r(y))$.*

Now, let us consider the news firm's problem, which is to choose the location of news y to maximize the indirect demand. The marginal effect of changing the news location y on the indirect demand is given by

$$\frac{\partial D_1}{\partial y}(y) = f(x_r)\frac{\partial x_r}{\partial y} - f(x_l)\frac{\partial x_l}{\partial y}.$$

We thus see that the optimal location of news y^* is dictated by the derivative of the limits of the interval of direct consumers who share the news, $x_l(y)$ and $x_r(y)$, with respect to y. Note that a consumer $\hat{x}(y) \in \{x_l(y), x_r(y)\}$ is indifferent between sharing the news and not. Thus, we have

$$u - \sigma^2(\hat{x}) - (y - \mu(\hat{x}))^2 = \tau. \tag{2}$$

Differentiating $\hat{x}$ with respect to y in (2) yields

$$\frac{\partial \hat{x}}{\partial y} = \frac{2\,(y - \mu(\hat{x}))}{2\mu_x(\hat{x})\,(y - \mu(\hat{x})) - \sigma_x^2(\hat{x})}. \tag{3}$$

The next two lemmas provide some intuitive results.

Lemma 3. *$x_l(y)$ is strictly increasing in $y \in [\underline{y}, 1]$ and $x_r(y)$ is strictly increasing in $y \in [0, \overline{y}]$.*

From Lemma 3, it follows that, under Assumptions 1-4, the breadth-maximizing news location y^* always belongs to the interval $[\underline{y}, \overline{y}]$. To see this, consider first $y < \underline{y}$. In this case,

$x_l(y) = 0$ and $D_1(y) = F(x_r(y))$. Then, $x_r(y)$ strictly increasing in $[0, \underline{y}]$ implies that $y^* \geq \underline{y}$. Analogously, it is easy to see that $x_l(y)$ strictly increasing in $[\overline{y}, 1]$ implies that $y^* \leq \overline{y}$.

From the symmetry of $F(\cdot)$ and $\tilde{g}(\cdot; x)$, we have the following lemma:

Lemma 4. *For any x and y,*

1. $\mu(x) = 1 - \mu(1-x)$;
2. $\sigma^2(x) = \sigma^2(1-x)$;
3. $x_r(y) = 1 - x_l(1-y)$.

We have the following result regarding (no) polarization of news:

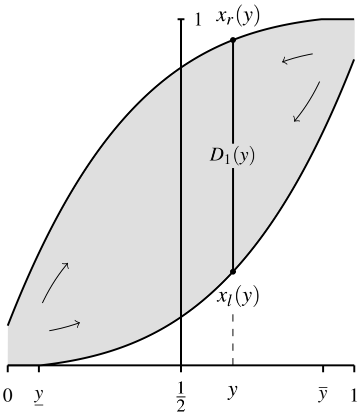
Proposition 2. *Assume that the density of direct readers is such that $f'(x) \geq 0$ for $x \in [0, 1/2)$.*

1. *$x_l(y)$ being strictly convex for all $y \in [\underline{y}, \overline{y}]$ is a sufficient condition for no polarization (i.e., $y^* = 1/2$) to be the unique maximizer of the breadth of news sharing.*
2. *$x_l(y)$ being concave enough for all $y \in [\underline{y}, \overline{y}]$ is a sufficient condition for the limit polarization strategy (i.e., $y^* = \underline{y}$ or $y^* = \overline{y}$) to be the unique maximizer of the breadth of news sharing. If $F(x)$ is uniformly distributed, $x_l(y)$ being strictly concave for all $y \in [\underline{y}, \overline{y}]$ is a sufficient condition for the limit polarization strategy to be the unique maximizer of the breadth.*

Remark 1. *The assumption of weakly increasing density of direct readers ensures a greater mass of direct consumers around the center, which makes no polarization easier to support. Namely in Proposition 2.1 strict convexity is enough to support no polarization whereas in Proposition 2.2 "enough" concavity is required to support the limit polarization. We also note that in the case of the opposite assumption of weakly decreasing density, the reverse result holds: in Proposition 2.1 "enough" convexity is required whereas in Proposition 2.2 strict concavity is enough.*

Figure 1 graphically explains Proposition 2 when $F(x)$ is uniformly distributed.

Although the proposition is general, it is stated in terms of convexity or concavity of $x_l(y)$, the meaning of which is hard to grasp. Therefore, we consider the case of a uniform F and try to understand the properties of $\mu(x)$ and $\sigma^2(x)$ that generate (no) polarization.



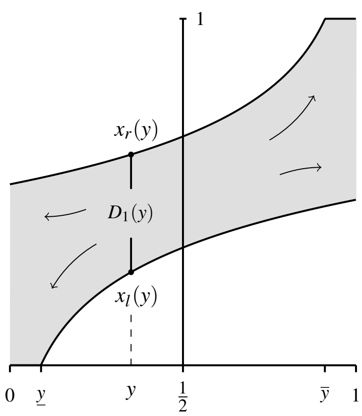


Figure 1: On the left, a convex $x_l(y)$ implies that indirect demand is maximized with no polarization, $y^* = 1/2$. On the right, a concave $x_l(y)$ implies that indirect demand is maximized with limit polarization, $y^* = \underline{y}$ or $y^* = \overline{y}$.

Suppose that F is uniformly distributed. Consider $x_l \in (0, 1/2)$. It then follows from equation (3) that

$$\frac{\partial^2 x_l}{\partial y^2} = \frac{2(\sigma_x^2)^2 - (2\mu_{xx}(y-\mu) - \sigma_{xx}^2)\,4(y-\mu)^2}{(2\mu_x(y-\mu) - \sigma_x^2)^3}.$$

Note that from Lemma 3, x_l is strictly increasing in y. This together with Lemma 2 implies from (3) that $(2\mu_x(y-\mu) - \sigma_x^2)$ is strictly positive at $x = x_l(y)$. From this equation, we can analyze the following cases:

- Case 1: $\sigma^2(x)$ constant (i.e., no variance effect).

  $$\frac{\partial^2 x_l}{\partial y^2} = -\frac{\mu_{xx}}{\mu_x^3}.$$

  Polarization occurs if $\mu(x)$ is convex on $[0, 1/2]$.

- Case 2: Both $\mu(x)$ and $\sigma^2(x)$ are linear (i.e., no second derivative effect).

  $$\frac{\partial^2 x_l}{\partial y^2} = \frac{2(\sigma_x^2)^2}{(2\mu_x\,(y-\mu) - \sigma_x^2)^3} > 0.$$

  Polarization never occurs.

- Case 3: $\mu(x)$ is convex and $\sigma^2(x)$ is linear on $[0, 1/2]$.

  $$\frac{\partial^2 x_l}{\partial y^2} = \frac{2(\sigma_x^2)^2 - 8\mu_{xx}(y-\mu)^3}{(2\mu_x\,(y-\mu) - \sigma_x^2)^3}.$$

  Polarization occurs if $\mu(x)$ is convex enough on $[0, 1/2]$.

- Case 4: $\mu(x)$ is linear and $\sigma^2(x)$ is concave on $[0, 1/2]$.

$$\frac{\partial^2 x_l}{\partial y^2} = \frac{2(\sigma_x^2)^2 + 4\sigma_{xx}^2(y-\mu)^2}{\left(2\mu_x\,(y-\mu) - \sigma_x^2\right)^3}.$$

  Polarization occurs if $\sigma^2(x)$ is concave enough on $[0, 1/2]$.

Table 1 provides the results for various cases.

	Constant variance	Concave variance	Linear variance	Convex variance
Concave mean	No	Yes, if variance concave enough	No	No
Linear mean	No	Yes, if variance concave enough	No	No
Convex mean	Yes	Yes, if mean convex enough or variance concave enough	Yes, if mean convex enough	Yes, if mean convex enough

Table 1: Cases where polarization occurs when f is uniform.

Summarizing, we obtain our central result.

Proposition 3. *Assume that $F(x)$ is uniformly distributed.*

1. *No polarization (i.e., $y^* = 1/2$) occurs if (i) the mean $\mu(x)$ is concave enough or the variance $\sigma^2(x)$ is convex enough for $x \in [0, 1/2]$, or (ii) both the mean $\mu(x)$ and variance $\sigma^2(x)$ are linear for $x \in [0, 1/2]$.*

2. *The limit polarization (i.e., $y^* = \underline{y}$ or $y^* = \overline{y}$) occurs if the mean $\mu(x)$ is convex enough or the variance $\sigma^2(x)$ is concave enough for $x \in [0, 1/2]$.*

We provide an intuition for why a convex mean $\mu(x)$ for $x \in [0, 1/2]$ can lead to polarization. Note first that by Assumption 4, $\mu(x)$ is increasing in x. Thus, a convex $\mu(x)$ implies that it increases in an increasing way for $x \in [0, 1/2]$. This means that the mean followers of two left-wing extremists with some distance are more closely located than those of two moderates with the same distance (see Figure 2). Therefore, the benefit from news sharing $B(x, y)$ decreases more slowly when x increases or decreases from x' satisfying $\mu(x') = y$ if the news is located close to the extremists than if it is located close to the moderates. This leads to the limit polarization.

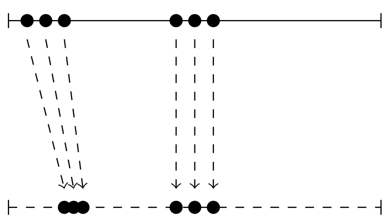


Figure 2: Convexity of $\mu(x)$ on $[0, 1/2]$ means that the mean followers of extremists are more closely located to each other than those of moderates.

To provide an intuition for why a concave variance $\sigma^2(x)$ for $x \in [0, 1/2]$ can lead to polarization, suppose that $\sigma^2(x)$ is increasing and concave for $x \in [0, 1/2]$. This means that the variance decreases as one moves away from the center, and sharply towards the extremes. Now, consider $y < 1/2$, and examine what happens when y further moves to the left. This movement will induce some left-wing extremists to start to share the news while inducing some moderates to stop sharing it. However, the gain in extremists will outweigh the loss in moderates. This is because, as per Lemma 1, a direct consumer with more homogeneous (i.e., lower variance) followers is more willing to share the news, all else equal. Because extremists have steeply more homogeneous followers, the leftward shift in the news ensures a substantial gain in extremists who are willing to share it compared to the loss of moderates. This logic implies that the limit polarization is optimal.

Let us now examine the effect of the distribution function of direct consumers. As long as the following function

$$\phi(y) = f(x_l(y)) \frac{\partial x_l(y)}{\partial y}$$

is increasing, it leads to $\frac{\partial D_1}{\partial y} > 0$ for $y < 1/2$ and $\frac{\partial D_1}{\partial y} < 0$ for $y > 1/2$, resulting in no polarization as the breadth-maximizing strategy. Analogously, ϕ decreasing leads to the limit polarization as the optimal strategy.

The derivative of ϕ is given by

$$f'(x_l) \left(\frac{\partial x_l}{\partial y} \right)^2 + f(x_l) \frac{\partial^2 x_l}{\partial y^2}.$$

Previously, we fixed the sign of f' to be nonnegative on $[0, 1/2)$. Now, note that $f'(x) \geq 0$ (resp., ≤ 0) generates a force toward no polarization (resp., the limit polarization). The U.S. experienced a dramatic increase in the polarization of partisan preferences over the past 40 years (Lazer et al., 2018). To the extent that this implies that f' is more negative over

$[0, 1/2)$, then this increase in the polarization of preferences makes news polarization more likely in our model.

3.2 Links to homophily

We here interpret Proposition 3 in terms of homophily. For this purpose, we first introduce a definition of homophily and show that interpreting the proposition in terms of this homophily measure can lead to some surprising predictions: against conventional wisdom, higher (lower) homophily at the center compared to the extremes can lead to (no) polarization. More generally, we show that it is the curvature, rather than the slope, of homophily that determines the nature of news polarization.

Definition 3. *Homophily at x is the negative mean square distance between a direct consumer at x and their followers:*

$$H(x) \equiv -\int_0^1 (x-z)^2 \tilde{g}(z;x)dz.$$

Similar to Lemma 1, we can write

$$H(x) = -(x-\mu(x))^2 - \sigma^2(x).$$

Namely, the homophily at x is sum of the negative squared distance between a direct consumer and their mean follower and the negative variance of followers.

To fix ideas, first consider the case in which the mean and the variance are linear. For instance, $\mu(x) = a + bx$ with $0 \leq a < 1/2$ and $b > 0$ and $\sigma^2(x) = cx + d$ with $c > 0$ and $d > 0$. Then, $H(x)$ strictly decreases with x in $[0, 1/2]$ for a small enough, meaning that homophily increases as x moves from the center to either extreme. However, from Proposition 3, we know that no polarization arises in this case no matter the slope of the homophily. Hence, we can conclude that increasing homophily when x moves from the center to extremes does not imply news polarization.

Consider now the case in which the mean is linear as before but the variance is non-linear and decreasing. In this case, $H(x)$ strictly increases with x in $[0, 1/2]$ for b close enough to 1. However, from Proposition 3, if the variance is concave enough, polarization occurs. As the second derivative of $H(x)$ is given by $H_{xx}(x) = -\sigma^2_{xx}(x) - 2(1-b)^2$ in this case, concave variance implies convex homophily for b close to 1. More generally, we show that what matters for (no) polarization is the convexity and the concavity of homophily as the next proposition states. Like Proposition 3, this results from Proposition 2.

Proposition 4. *Assume that F is uniformly distributed. Then:*

1. *Decreasing homophily $H(x)$ on $[0, \frac{1}{2}]$ can lead to no polarization. Conversely, increasing homophily on $[0, \frac{1}{2}]$ can lead to limit polarization if $H(x)$ is convex enough.*
2. *More generally, if $H(x)$ is sufficiently convex on $[0, \frac{1}{2}]$, there is limit polarization; if $H(x)$ is sufficiently concave on $[0, \frac{1}{2}]$, there is no polarization.*

4 Competition

Here, we consider competition between two media firms, L and R. Each firm i (with $i = L, R$) chooses the ideological position of its news y_i. Without loss of generality, we can consider $y_L \leq y_R$ in equilibrium. We consider no targeting and assume that all direct consumers receive both news items, but each of them shares at most one news item. A direct consumer shares the news that generates the larger average benefit conditional on it being larger than the attention tax.

Let $[x_l(y_L), x_r(y_L)]$ (resp. $[x_l(y_R), x_r(y_R)]$) be the interval of locations of direct consumers who prefer sharing the news from firm L (resp. firm R) over not sharing any news. The functions $x_l(\cdot)$ and $x_r(\cdot)$ are determined in the same way as in the baseline model. Suppose that $x_l(y_R) < x_r(y_L)$. Then a direct consumer is indifferent between sharing L's news and sharing R's news if she is located at $x_m = x_m(y_L, y_R)$ such that $B(x_m, y_L) = B(x_m, y_R)$, which is equivalent to $\mu(x_m) = (y_L + y_R)/2$. Therefore, the firms' respective indirect demands are

$$D_L(y_L, y_R) = F\left(x_m(y_L, y_R)\right) - F\left(x_l(y_L)\right)$$

and

$$D_R(y_L, y_R) = F\left(x_r(y_R)\right) - F\left(x_m(y_L, y_R)\right).$$

Let (y_L^*, y_R^*) represent the equilibrium news locations under competition between the two media firms. The next lemma shows an intuitive result:

Lemma 5. *Under competition between the two media firms, if an equilibrium exists, we must have $y_L^* \leq 1/2 \leq y_R^*$.*

If the attention tax is low enough, we have $x_l(1/2) = 0$ and $x_r(1/2) = 1$. In that case, as in the baseline model, we have no polarization.

Proposition 5. *Under competition between the two media firms, if the attention tax is low enough that even the most extreme consumer is willing to share ideologically neutral news, there is a unique equilibrium in which both firms adopt the no-polarization strategy, i.e.,* $y_L^* = y_R^* = 1/2$*.*

The above proposition is similar to the result obtained by Gabszewicz, Laussel and Sonnac (2001) in a Hotelling model in which two newspapers compete in prices. When the nonnegative pricing constraint of newspapers binds due to high advertising revenue, they find that both newspapers locate at the middle. However, they consider neither social media nor news sharing.

We now consider the case of $\tau > B(x = 0, y = 1/2)$ and study when it is optimal for each firm to adopt the limit polarization. Suppose that F is uniform. The next proposition shows that under mild conditions, the limit polarization in the baseline model without competition implies the limit polarization under competition between the two media firms.

Proposition 6. *Suppose that F is uniformly distributed. The limit polarization in the baseline model without competition implies the limit polarization under competition between the two media firms $y_L^* = \underline{y}$ and $y_R^* = \overline{y}$ whenever, for $x \in [0, 1/2]$,*

1. *$\mu(x)$ is convex and $\sigma_x^2 \leq 0$ or $\sigma_x^2 > 0$ but small; or*

2. *$\mu(x)$ is linear and $\sigma^2(x)$ is concave but with a slope not very negative.*

Note first that the conditions in the proposition such as convex mean and concave variance are consistent with the limit polarization in the baseline model without competition (see Proposition 3). To provide an intuition for the result, suppose that firm L chooses some $y_L < \overline{y}$ given $y_R = \overline{y}$. Then, the interval of direct consumers who share L's news is given by $[x_l(y_L), x_m(y_L, \overline{y})]$. Compare this interval with the interval of sharers $[x_l(y_L), x_r(y_L)]$ for a monopolist choosing the same $y = y_L$. Consider now moving y_L slightly to the left. Then, the induced decrease in $x_l(y_L)$ is the same in both intervals. However, the induced decrease in $x_m(y_L, \overline{y})$ is typically smaller than the one in $x_r(y_L)$. This is because the marginal consumer under competition, who is located at $x_m(y_L, \overline{y})$, has an outside option of sharing news from

R of which the benefit increases with her location, whereas the marginal consumer in the baseline model without competition, who is located at $x_r(y_L)$, has no outside option, which is akin to having an outside option that does not depend on her location. As y_L moves to the left, the location of the marginal consumer $x_m(y_L, \overline{y})$ also moves to the left, which in turn makes less attractive the outside option of sharing news from R (see Figure 3). This can induce the reduction in $x_m(y_L, \overline{y})$ under competition to be smaller than the reduction in $x_r(y_L)$ in the monopolist case, leading to polarization under competition.

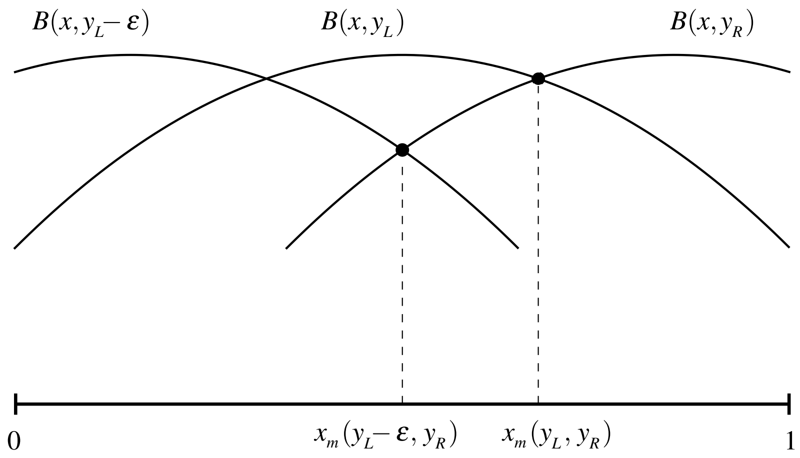


Figure 3: In the figure, firm L moves the location of its news to the left given the location of R's news.

5 Extensions

In this section, we provide three extensions of the baseline model.

5.1 Targeting

Here, we extend the baseline model by enabling the media firm to target an interval of direct readers. For instance, the Macedonian teenagers, mentioned in the introduction, purchased bogus Facebook accounts and used them to target certain profiles of users to spread their fake news.

Suppose that the media firm can target direct consumers belonging to an interval of given length $l \in [0, 1)$ to send its news. Suppose also that direct consumers are uniformly distributed. Hence, the media firm now should choose not only the location of its news y but also the target interval $[a, a+l] \subset [0,1]$ by choosing a. We show that it is a weakly dominant strategy to choose the optimal location without targeting (i.e., $y = y^*$) and $a = x_l(y^*)$.

Proposition 7 (Targeting Strategy)**.** *Suppose that the media firm can choose both the news location y and the target interval of direct consumers $[a, a+l] \subset [0,1]$ by choosing a where $l \in [0,1)$ is exogenously given. Suppose that F is uniformly distributed. It is a weakly dominant strategy for the firm to choose $y = y^*$ and $a = x_l(y^*)$.*

Proof. We need to distinguish two cases: either $a + l \leq x_r(y^*)$ or $a + l > x_r(y^*)$. In the first case, all targeted consumers share the news and one cannot do better. In the second case, all consumers in the interval $[x_l(y^*), x_r(y^*)]$ are targeted and share the news and hence one cannot do better either. □

5.2 Resharing

Here, we extend the baseline model by adding one additional layer of communication: the indirect consumers can share news with their own followers. The utility from reading news is given by $U(x, y) = u - (x - y)^2$ as before. We maintain the assumption that each group of followers is distinct at any layer.

The profit of the firm is now given by

$$\pi(y) = \sum_{n=0}^{2} \alpha_n D_n(y)$$

where $D_n(y)$ denotes the demand from consumers at the n-th layer of communication and $\alpha_n \geq 0$ is a weight for the demand from consumers at the n-th layer with $n = 0, 1, 2$.

We below introduce a measure of connectedness between the first-layer followers and the second-layer followers. Note that we allow for the distribution of the former to be different from that of the latter.

Definition 4 (Connectedness between two layers of followers)**.** *Given an ideological segment $[a, a+l]$, let $c_2(a, l)$ denote the measure of 2nd layer consumers whose ideological locations belong to the interval $[a, a+l]$ and follow a first-layer consumer located in the same interval. We refer to $c_2(a, l)$ as the 2nd layer connectedness on the interval $[a, a+l]$. Additionally, we* *say that the connectedness is extremal (central) if this measure increases toward the extremes (resp., center), that is,*

$$\frac{\partial c_2}{\partial a}(a,l) = \begin{cases} < 0 \ (\textit{resp.}, \ > 0) & \textit{for } a \in [0, 1/2 - l/2) \\ > 0 \ (\textit{resp.}, \ < 0) & \textit{for } a \in (1/2 - l/2, 1 - l]. \end{cases}$$

We have the following result:

Proposition 8 (Resharing)**.** *Suppose that F is uniformly distributed.*

1. *No polarization in the baseline model implies no polarization in the extension with one additional layer if the connectedness is central.*

2. *The limit polarization in the baseline model implies the limit polarization in the extension with one additional layer if the connectedness is extremal.*

Asymmetries in the degree of homophily are observed on Twitter by Boutyline and Willer (2017). Using a data set of the entire Twitter network from 2009, the authors find that more extreme individuals tend to be more homophilous than more moderate ones. Then, our measure of connectedness is likely to be extremal and, according to the above proposition, the limit polarization in the baseline model implies the limit polarization in the extension.

5.3 The depth-maximization strategy

Here, we provide an extension of the baseline model that focuses on *the depth of news sharing*. For this purpose, we allow for the resharing of news following down many layers of communication. We maintain the assumption that each group of followers is distinct at any layer. In addition, we assume that any potential sharer located at x has a distribution of followers given by the density $\tilde{g}(z;x)$.

For the resharing to stop at some layer, we assume that the constant u depreciates when the layer of sharing increases. That is, the utility that a consumer located at x obtains from reading a news item located at y after it has been shared $n = 1, 2, \ldots$ times is

$$U_n(x,y) = \delta^{n-1}u - (x-y)^2, \quad \text{with } \delta \in (0,1),$$

and the benefit generated by sharing the news item one additional time is given by

$$B_{n+1}(x,y) = \int_0^1 \max\{U_{n+1}(z,y), 0\} \ \tilde{g}(z;x)\, dz.$$

For the sake of computational simplicity, we adopt a continuous version of the problem. Let $t \in [0, +\infty)$ be the time at which a news item located at y reaches an indirect consumer located at x. Then, a consumer's utility from reading the news and her benefit of resharing it will be given, respectively, by

$$U_t(x, y) = \delta^t u - (x - y)^2 \quad \text{and} \quad B_t(x, y) = \int_0^1 \max\{U_t(z, y), 0\}\, \tilde{g}(z; x)\, dz.$$

Definition 5 (Depth of News Sharing)**.** *We define the depth of news sharing as the maximum number of times that a news item is shared following down the hierarchical layers of communication.*

We study the depth-maximization strategy and allow the firm to target only one direct consumer without loss of generality. Hence, the firm chooses the location of the targeted consumer and the location of the news item. We have the following result:

Proposition 9 (Depth-Maximizing Strategy)**.** *Suppose that the attention tax is not small.*[6] *The media firm's optimal strategy to maximize the depth of news sharing is characterized as follows:*

1. *It targets the consumer located at x^*_{depth} whose $\sigma^2(x)$ is equal to $\min_{x \in [0,1/2]} \sigma^2(x)$.*
2. *It chooses the location of the news item $y^*_{depth} = \mu(x^*_{depth})$.*

Corollary 1. *Suppose that the attention tax is not small.*

1. *Depth maximization leads to some polarization as long as $\sigma^2(1/2) > \min_{x \in [0,1/2]} \sigma^2(x)$;*
2. *If $\sigma^2(x)$ is increasing on $[0, 1/2]$, it leads to $x^*_{depth} = 0$.*

The assumption that the attention tax is not small means that for consumers at the target location, as the news depreciates, the net benefit of sharing it becomes zero before the utility of reading it becomes zero. For a given consumer, her benefit from sharing a news item is maximized when the item has the same location as that of her mean follower. From equation (1), this leads to a benefit equal to $u - \sigma^2(x)$. Therefore, it is optimal to target the location of the direct consumer with the lowest variance $\sigma^2(x)$.

[6] The exact meaning of the attention tax not small is explained in the paragraph following this proposition.

If one considers that false political news tends to be hyperpartisan (Silverman et al., 2016), our results provide an explanation for the findings of Vosoughi, Roy and Aral (2018) that false political news diffused deeper and more broadly than true news. The authors suggest that the degree of novelty and the extent to which the news is emotionally charged may explain their findings. Our results suggest that their findings may also be explained by the structure of social networks: a news item appealing to a group of consumers who have very homogeneous ideological preferences tend to diffuse deeply.

6 Conclusion

We have studied how the distribution of followers' ideological preferences shapes a user's incentive to share news on social media and how this, in turn, does or not create incentives for a media firm to provide partisan content. In particular, we have focused on how the distribution of the mean and the variance of followers' ideological locations affect the ideological location of news. We have found that both a convex mean and a concave variance contribute to polarization when a media firm maximizes the breadth of news sharing. When reinterpreted in terms of the measure of homophily we introduce, this result implies that contrary to conventional wisdom, more homophily at the extremes can lead to no polarization while more homophily at the center can lead to polarization. More generally, what matters for polarization is the curvature, rather than the slope, of homophily. This implication can be tested empirically.

Although we assumed that both the distribution of direct consumers and that of followers are symmetric around $1/2$, in reality, more conservative individuals are more homophilous than more liberal ones (Boutyline and Willer (2017)). It would be interesting to extend our framework to incorporate such asymmetry.

We believe that the most interesting avenue for future research is to examine the role of news feed algorithms (Berman and Katona, 2020). In the case of Facebook, after a user decides to share news with her friends, the algorithm determines which subset of her friends will be exposed to the news. If a social media platform can employ an algorithm to maximize the amount of time users spend on the platform, will such an engagement-maximizing algorithm lead to more or less polarization of news?

Another interesting avenue for future research consists in incorporating consumers' actions such as voting and study how news consumption and polarization influences their actions (Galeotti and Mattozzi, 2011).

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Appendix

Proof of Proposition 1

The quasiconcavity of the benefit function implies that, for any y, $B(x, y) \geq \min\{B(0, y), B(1, y)\}$, for all x. Additionally, from the symmetry of the problem, $\max_{y\in[0,1]} \min\{B(0, y), B(1, y)\} = B(0, 1/2)$ $(= B(1, 1/2))$.

Therefore, if $\tau \leq B(0, 1/2)$, by locating its news at $y^* = 1/2$ the media firm can induce sharing from all direct consumers, thus maximizing the breadth of news sharing.

Proof of Lemma 2

Let us show that $y \geq \underline{y}$ implies $y > \mu(x_l(y))$ (the second statement follows analogously). To this end, assume $y > \underline{y}$. By part 1 of Assumption 4, $\mu(\cdot)$ is invertible, and thus showing $y > \mu(x_l(y))$ is equivalent to showing that $\mu^{-1}(y) > x_l(y)$. Since $y \geq \underline{y}$, $B(x_l(y), y) = \tau$, and thus by Assumption 2, to show $\mu^{-1}(y) > x_l(y)$ it is suffices to show $B(\mu^{-1}(y), y) > \tau$. It follows from Lemma 1 that

$$
\begin{aligned}
B(\mu^{-1}(y), y) &= u - \sigma^2(\mu^{-1}(y)) - (y - \mu(\mu^{-1}(y)))^2 \\
&= u - \sigma^2(\mu^{-1}(y)) \\
&> \tau,
\end{aligned}
$$

where the last inequality follows from part 2 of Assumption 4.

Proof of Lemma 3

Note that

$$\frac{\partial B(x,y)}{\partial x} = 2\mu_x(x)(y - \mu(x)) - \sigma_x^2(x).$$

In addition, from $B(\cdot, y)$ being quasiconcave and differentiable follows that

$$\left.\frac{\partial B(x,y)}{\partial x}\right|_{x=x_l(y)} > 0 \qquad \text{and} \qquad \left.\frac{\partial B(x,y)}{\partial x}\right|_{x=x_r(y)} < 0.$$

Taking this together with Lemma 2 and equation (3) implies that both $\partial x_l/\partial y$ and $\partial x_r/\partial y$ are positive whenever x_l and x_r satisfy equation (2).

Proof of Lemma 4

1 and 2 follow directly from our assumption that $\tilde{g}(z; x) = \tilde{g}(1 - z; 1 - x)$.

To prove 3, we start by noting that 1 and 2 imply that $B(x, y) = B(1 - x, 1 - y)$ for all $x, y \in [0, 1]$. From that, if $x_l(y)$ and $x_r(y)$ are solutions for $B(\hat{x}, y) = \tau$, then $1 - x_l(y)$ and $1 - x_r(y)$ are solutions for $B(\hat{x}, 1 - y) = \tau$. Finally, as $x_l(y) < x_r(y)$ implies $1 - x_l(y) > 1 - x_r(y)$, we have that $x_l(1 - y) = 1 - x_r(y)$.

Proof of Proposition 2

1. Take $y < 1/2$.

   (a) From Lemma 3 we have that x_l is strictly increasing. Then, $x_l(y) < x_l(1-y)$ for any $y < 1/2$. Additionally, we have that $x_l(y) < x_r(y) = 1 - x_l(1-y)$, i.e., the distance between $x_l(y)$ and 0 is shorter than the distance between $x_l(1-y)$ and 1. Hence, $f' \geq 0$ for $x \in [0, 1/2)$ and symmetry of f imply that $f(x_l(y)) \leq f(x_l(1-y)) = f(x_r(y))$;

   (b) x_l strictly convex implies $\frac{\partial x_l(y)}{\partial y} < \frac{\partial x_l(1-y)}{\partial y}$; and

   (c) From Lemma 4 we have $x_r(y) = 1 - x_l(1-y)$. Hence, $\frac{\partial x_r(y)}{\partial y} = \frac{\partial x_l(1-y)}{\partial y}$.

   From (a)-(c) and from the symmetry of f follows that

   $$\frac{\partial D_1}{\partial y}(y) = f(x_r(y))\frac{\partial x_r}{\partial y}(y) - f(x_l(y))\frac{\partial x_l}{\partial y}(y) > 0.$$

   It is straightforward to verify that $y > 1/2$ implies $\frac{\partial D_1}{\partial y}(y) < 0$. Therefore, the breadth is maximized at $y^* = 1/2$.

2. For any $y < \underline{y}$, we have that $x_l(y) = 0$ and

   $$\frac{\partial D_1}{\partial y}(y) = f(x_r(y))\frac{\partial x_r}{\partial y}(y) \geq 0.$$

   On the other hand, for any $\underline{y} < y < 1/2$, we have that if $x_l(y)$ is concave enough such that

   $$\frac{\partial^2 x_l}{\partial y^2} < -\frac{f'(x_l)}{f(x_l)}\left(\frac{\partial x_l}{\partial y}\right)^2,$$

   then the product $f(x_l)\frac{\partial x_l}{\partial y}$ is decreasing in y, and we have that

   $$\frac{\partial D_1}{\partial y}(y) = f(x_l(1-y))\frac{\partial x_l}{\partial y}(1-y) - f(x_l(y))\frac{\partial x_l}{\partial y}(y) < 0.$$

   Hence, the limit polarization strategy is optimal.

Proof of Proposition 3

When direct readers are uniformly distributed, it follows that

$$\begin{aligned}\frac{\partial D_1}{\partial y}(y) &= \frac{\partial x_r}{\partial y}(y) - \frac{\partial x_l}{\partial y}(y) \\ &= \frac{\partial x_l}{\partial y}(1-y) - \frac{\partial x_l}{\partial y}(y).\end{aligned}$$

Therefore, no polarization (respectively, limit polarization) occurs if $x_l(y)$ is convex (respectively, concave) on $[\underline{y}, \overline{y}]$. From equation (3) we have that

$$\frac{\partial^2 x_l}{\partial y^2} = \frac{2\left(\sigma_x^2(x_l)\right)^2 - \left(2\mu_{xx}(x_l)\left(y-\mu(x_l)\right) - \sigma_{xx}^2(x_l)\right)4\left(y-\mu(x_l)\right)^2}{\left(2\mu_x(x_l)\left(y-\mu(x_l)\right) - \sigma_x^2(x_l)\right)^3}.$$

Hence,

$$\frac{\partial^2 x_l}{\partial y^2} \gtreqless 0 \quad \text{if and only if} \quad 2\mu_{xx}(x_l)\left(y-\mu(x_l)\right) - \sigma_{xx}^2(x_l) \lesseqgtr \frac{1}{2}\left(\frac{\sigma_x^2(x_l)}{y-\mu(x_l)}\right)^2,$$

and the proposition follows.

Proof of Proposition 4

The proof of 1 is omitted as it is shown in the main text. We prove 2 in what follows. It follows from Assumption 3 (namely, that $\mu(x)$ is strictly increasing in x) that $x_l(\cdot)$ is a bijection. Thus, we define the inverse function $y_l(\cdot) \equiv x_l^{-1}(\cdot)$. Thus, for all $x \in [0,1]$,

$$u - \sigma^2(x) - (y_l(x) - \mu(x))^2 = \tau.$$

Differentiating twice with respect to x, we obtain

$$\sigma_{xx}^2(x) + 2(y_l(x) - \mu(x))(\frac{d^2 y_l(x)}{dx^2}) - 2(y_l(x) - \mu(x))\mu_{xx}(x) + 2(\frac{dy_l(x)}{dx} - \mu_x(x))^2 = 0. \quad (4)$$

Further note that

$$H_{xx}(x) = -\sigma_{xx}^2(x) + 2\mu_{xx}(x)(x - \mu(x)) - 2(1 - \mu_x(x))^2. \quad (5)$$

It follows from (4) and (5) that

$$H_{xx}(x) + 2(y_l(x) - \mu(x))(\frac{d^2 y_l(x)}{dx^2}) + 2(\frac{dy_l(x)}{dx} - 1)^2 + 4(\frac{dy_l(x)}{dx} - 1)(1 - \mu_x(x)) - 2(y_l(x) - x)\mu_{xx}(x) = 0.$$

Let

$$\alpha(x) \equiv 2(\frac{dy_l(x)}{dx} - 1)^2 + 4(\frac{dy_l(x)}{dx} - 1)(1 - \mu_x(x)) - 2(y_l(x) - x)\mu_{xx}(x).$$

Now note:

$$H_{xx}(x) + 2(y_l(x) - \mu(x))(\frac{d^2 y_l(x)}{dx^2}) + \alpha(x) = 0. \tag{6}$$

It follows from Assumption 4 that $y_l(x) - \mu(x) > 0$ for all x.

Now suppose H is sufficiently convex in the sense that $H_{xx}(x) > \alpha(x)$ for all x. It follows from (6) that $\frac{d^2 y_l(x)}{dx^2} < 0$ for all $x \in [0, \frac{1}{2}]$. Hence, $\frac{d^2 x_l(y)}{dy^2} < 0$ for all $y \in [\underline{y}, \frac{1}{2}]$. It then follows from reasoning presented in the proof of Proposition 3 that there is limit polarization.

One can analogously show that if $H_{xx}(x) < \alpha(x)$ for all x, there is no polarization.

Proof of Lemma 5

Let (y_L^*, y_R^*) be an equilibrium with $y_L^* \leq y_R^*$.

We prove the result by contradiction. Suppose, for example, that $y_L^* \leq y_R^* < 1/2$. Then, $D_L(y_L^*, y_R^*) = x_m(y_L^*, y_R^*) - x_l(y_L^*)$ and

$$\begin{aligned} D_L(1 - y_L^*, y_R^*) &= x_r(1 - y_L^*) - x_m(1 - y_L^*, y_R^*) \\ &= [1 - x_l(y_L^*)] - x_m(1 - y_L^*, y_R^*) \\ &= [1 - x_m(1 - y_L^*, y_R^*)] - x_l(y_L^*). \end{aligned}$$

As (y_L^*, y_R^*) is an equilibrium, we must have $D_L(y_L^*, y_R^*) \geq D_L(1 - y_L^*, y_R^*)$ or, equivalently, $x_m(y_L^*, y_R^*) \geq 1 - x_m(1 - y_L^*, y_R^*)$.

As the mean function μ is increasing and that we have

$$\mu\left(x_m(y_L^*, y_R^*)\right) = \frac{y_L^* + y_R^*}{2}$$

and

$$\begin{aligned} \mu\left(1 - x_m(1 - y_L^*, y_R^*)\right) &= 1 - \mu\left(x_m(1 - y_L^*, y_R^*)\right) \\ &= 1 - \frac{1 - y_L^* + y_R^*}{2} = \frac{y_L^* + (1 - y_R^*)}{2}, \end{aligned}$$

$x_m(y_L^*, y_R^*) \geq 1 - x_m(1 - y_L^*, y_R^*)$ if and only if $y_R^* \geq 1 - y_R^*$, that is, $y_R^* \geq 1/2$, which contradicts our initial hypothesis that $y_R^* < 1/2$.

Proof of Proposition 5

Clearly, $y_L^* = y_R^* = 1/2$ is an equilibrium since by deviating from the center a firm does not increases the number of sharing from extremists (since all of them were already sharing) while it loses sharing from some moderates who now prefer to share the news of its competitor.

In any other tentative equilibrium where one firm, say firm L, plays $y_L \neq 1/2$, the best reply for firm R is to chose a location y_R between y_L and $1/2$, the closest possible of y_L. But since whenever $y_R \neq 1/2$ then the best reply of firm L is to choose location y_L between y_R and $1/2$, the closest possible of y_R, the only possible equilibrium is $y_L^* = y_R^* = 1/2$.

Proof of Proposition 6

Limit polarization occurs in the baseline model without competition when we have

$$\frac{\partial D_1}{\partial y}(y) = \frac{\partial x_r}{\partial y}(y) - \frac{\partial x_l}{\partial y}(y) = \begin{cases} < 0 & \text{for } y \in (\underline{y}, 1/2) \\ > 0 & \text{for } y \in (1/2, \overline{y}). \end{cases}$$

Under competition between the two media firms, we consider the deviation of the L firm given that the R firm chooses $y_R = \overline{y}$. Then we have

$$\frac{\partial D_L}{\partial y_L}(y, \overline{y}) = \frac{\partial x_m}{\partial y_L}(y, \overline{y}) - \frac{\partial x_l}{\partial y_L}(y).$$

Given that the rival is located at $\overline{y}$, we have $1/2 \leq x_m(y, \overline{y}) \leq x_r(y)$ for any $y \in \left[\underline{y}, \overline{y}\right]$.

Claim 1. $\frac{\partial D_L}{\partial y_L}(y, \overline{y}) \leq \frac{\partial D_1}{\partial y}(y)$ for any $y \in (\underline{y}, 1/2)$.

This claim implies $|\frac{\partial D_L}{\partial y_L}(y, \overline{y})| \geq |\frac{\partial D_1}{\partial y}(y)| > 0$ for $y \in (\underline{y}, 1/2)$. Hence, if the L firm moves to the left in $(\underline{y}, 1/2)$, its indirect demand increases more than the increase in indirect demand experienced by the monopolist from the same change in y. Therefore, it is optimal for the former to choose $\underline{y}$ when we consider $y \leq 1/2$.

Claim 2. $\frac{\partial D_1}{\partial y}(y) \geq \frac{\partial D_L}{\partial y}(y, \overline{y})$ for any $y \in (1/2, \overline{y})$.

This claim implies that if the L firm moves to the right in $(1/2, \overline{y})$, its indirect demand increases less than the increase in indirect demand experienced by the monopolist from the same change in y.

As the monopolist's indirect demand is symmetric around $1/2$ and is maximized when it chooses $y = \underline{y}$ or $y = \overline{y}$, Claim 1 and 2 imply that it is optimal for the L firm to choose $\underline{y}$ (see Figure 4).

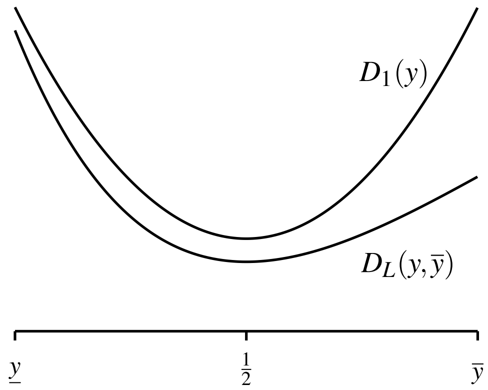

Figure 4: Indirect demand functions of the monopolist and the duopolist. The monopolist has a larger loss (respectively, gain) than the duopolist for $y < 1/2$ (respectively, $y > 1/2$).

Claim 1 is equivalent to $\frac{\partial x_m}{\partial y_L}(y,\overline{y}) \leq \frac{\partial x_r}{\partial y}(y)$ for $y \in (\underline{y}, 1/2)$ and Claim 2 is equivalent to $\frac{\partial x_r}{\partial y}(y) \geq \frac{\partial x_m}{\partial y_L}(y,\overline{y})$ for $y \in (1/2, \overline{y})$.

Hence, to prove both claims, it is enough to show $\frac{\partial x_m}{\partial y_L}(y,\overline{y}) \leq \frac{\partial x_r}{\partial y}(y)$, for any $y \in [\underline{y}, \overline{y}]$.

Now, we have

$$\frac{\partial x_m}{\partial y_L}(y,\overline{y}) = \frac{1}{2\mu_x(x_m)}$$

and

$$\frac{\partial x_r}{\partial y}(y) = \frac{2(\mu(x_r) - y)}{2\mu_x(x_r)\,(\mu(x_r) - y) + \sigma_x^2(x_r)}.$$

Therefore, $\partial x_m/\partial y_L \leq \partial x_r/\partial y$ if and only if

$$2\,(2\mu_x(x_m) - \mu_x(x_r))\,(\mu(x_r) - y) \geq \sigma_x^2(x_r).$$

1. In case we have μ convex on $[0, 1/2]$ and σ^2 constant on $[0, 1/2]$:

   From $\mu(x)$ convex on $[0, 1/2]$, and from symmetry, follows that $\mu(x)$ is concave on $[1/2, 1]$. Therefore, for any $y \in [\underline{y}, \overline{y}]$

   $$\mu_x\,(x_m(y,\overline{y})) > \mu_x\,(x_r(y))$$

   which is equivalent to

   $$\frac{1}{\mu_x\,(x_m(y,\overline{y}))} < \frac{1}{\mu_x\,(x_r(y))}.$$

   Therefore,

   $$\frac{\partial x_m}{\partial y_L}(y,\overline{y}) = \frac{1}{2\mu_x\,(x_m(y,\overline{y}))} < \frac{1}{\mu_x\,(x_m(y,\overline{y}))} < \frac{1}{\mu_x\,(x_r(y))} = \frac{\partial x_r}{\partial y}(y).$$

From the above argument, it is clear that $\partial x_m/\partial y_L \leq \partial x_r/\partial y$ holds when $\sigma_x^2 < 0$ on $[0, 1/2]$ (or, equivalently, $\sigma_x^2 > 0$ on $[1/2, 1]$) as well.

Furthermore, as we have some strict gap created by comparison between $\frac{1}{2\mu_x(x_m)}$ and $\frac{1}{\mu_x(x_r)}$, the result carries out even if $\sigma_x^2 > 0$ but small on $[0, 1/2]$.

2. In case we have μ linear and σ^2 concave on $[0, 1/2]$:

   Let $\mu(x) = mx$, for $x \in [0, 1/2)$. Hence, $\mu(x) = 1 - m(1-x)$, for $x \in (1/2, 1]$.

   In that case, $\frac{\partial x_m}{\partial y_L}(y, \overline{y}) \leq \frac{\partial x_r}{\partial y}(y)$ requires $\sigma_x^2(x_r) \leq 2m\left(\mu(x_r) - y\right)$, that is, the slope of σ^2 cannot be very negative on $[0, 1/2]$ (equivalently, not very positive on $[1/2, 1]$).

Both in the case (i) σ^2 is linear and μ is either linear or concave and in the case (ii) μ is linear and σ^2 is convex, the limit polarization is not optimal in the baseline model without competition. Therefore, those cases are ruled out from our analysis.

Proof of Proposition 8

Let us start by showing that the optimum y cannot be more extreme than y^*.

For $y \leq y^*$ we have

$$D_2(y) = \int_0^{x_r(y)} \tilde{G}_2\left(x_r(y); x\right)\, dx,$$

where $\tilde{G}_2(\cdot; x)$ represents the cumulative distribution of the second-layer followers of a first-layer consumer located at x. Thus, as $x_r(y)$ is increasing in y, $D_2(y)$ is increasing in y for any $y \leq y^*$.

Suppose that the limit polarization is optimal in the baseline model with one layer of followers. Then, for any $y \in (y^*, 1/2]$, we have that $x_r(y) - x_l(y) < x_r(y^*) - x_l(y^*) = x_r(y^*)$. It is enough to verify that the limit polarization maximizes the second-layer indirect demand. We have

$$D_2(y^*) = \int_0^{x_r(y^*)} \tilde{G}_2(x_r(y^*); x)\, dx = c_2\left(0, x_r(y^*)\right).$$

Thus, if the connectedness is extremal, for any $y \in (y^*, 1/2]$, we have

$$D_2(y^*) = c_2(0, x_r(y^*)) > c_2(x_l(y), x_r(y^*)).$$

In addition, the following inequality holds

$$c_2(x_l(y), x_r(y^*)) \geq D_2(y) = \int_{x_l(y)}^{x_r(y)} \tilde{G}_2(x_r(y); x) - \tilde{G}_2(x_l(y); x)\, dx$$

since we have

$$
\begin{aligned}
c_2(x_l(y), x_r(y^*)) &= \int_{x_l(y)}^{x_l(y)+x_r(y^*)} \tilde{G}_2(x_l(y) + x_r(y^*); x) - \tilde{G}_2(x_l(y); x)\, dx \\
&= \int_{x_r(y)}^{x_l(y)+x_r(y^*)} \tilde{G}_2(x_l(y) + x_r(y^*); x) - \tilde{G}_2(x_l(y); x)\, dx \\
&\quad + \int_{x_l(y)}^{x_r(y)} \tilde{G}_2(x_l(y) + x_r(y^*); x) - \tilde{G}_2(x_r(y); x)\, dx + D_2(y) \\
&\geq D_2(y)
\end{aligned}
$$

where the last inequality comes from the non-negativity of the terms with integrals. This proves that the limit polarization maximizes the second-layer indirect demand.

The proof for the case of no polarization is analogous.

Proof of Proposition 9

Let t^* denote the depth of sharing the news located at y. Since, by assumption, a consumer located at x reshares the news located at y whenever both $U_{t(y)}(x, y) \geq 0$ and $B_{t^*}(x, y) \geq \tau$, if the attention tax is not small, the latter inequality is the first to bind. Therefore,

$$
\begin{aligned}
B_{t^*}(x, y) &= \int_0^1 U_{t^*}(z, y)\, \tilde{g}(z; x)\, dz \\
&= \delta^{t^*} u - \sigma^2(x) - (y - \mu(x))^2 .
\end{aligned}
$$

And from $B_{t(y)}(x, y) = \tau$ follows that

$$
\delta^{t^*} u - \tau = \sigma^2(x) + (y - \mu(x))^2 .
$$

As the L.H.S. of the equation above is decreasing in t^*, the depth is maximized when the R.H.S. is minimized. The result follows.